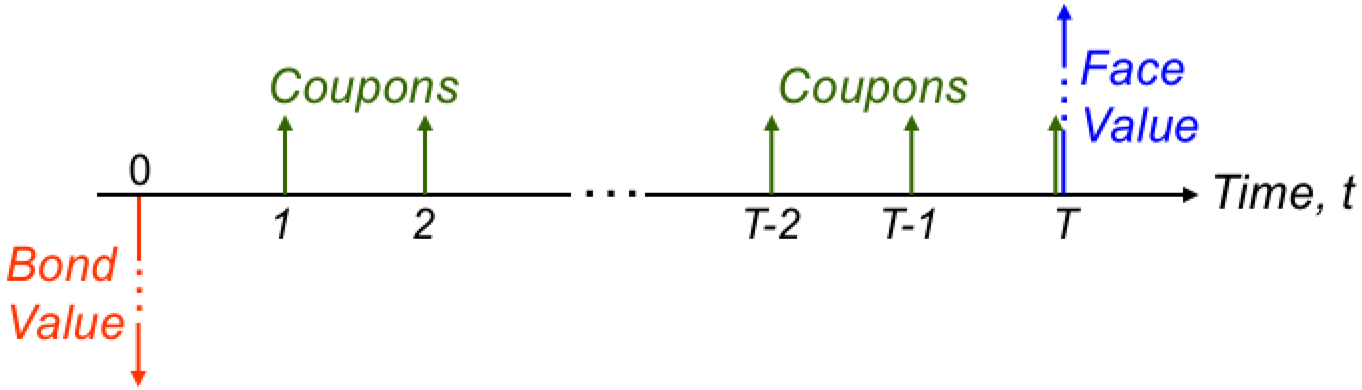
Bond Pricing

What Is a Bond?

- A bond is a security that obligates the issuer to make specified payments to the holder over a period of time



Bond Value = Amount Paid **by** the Investor to Buy the Bond

Coupons = Interest Paid **to** the Investor for Holding the Bond

Face Value = Amount Paid **to** the Investor at Maturity (*T*)

Bond Pricing

- Assuming that the discount rate (r) is constant:

$$\textit{Bond Value} = PV(\textit{Coupons}) + PV(\textit{Face Value})$$

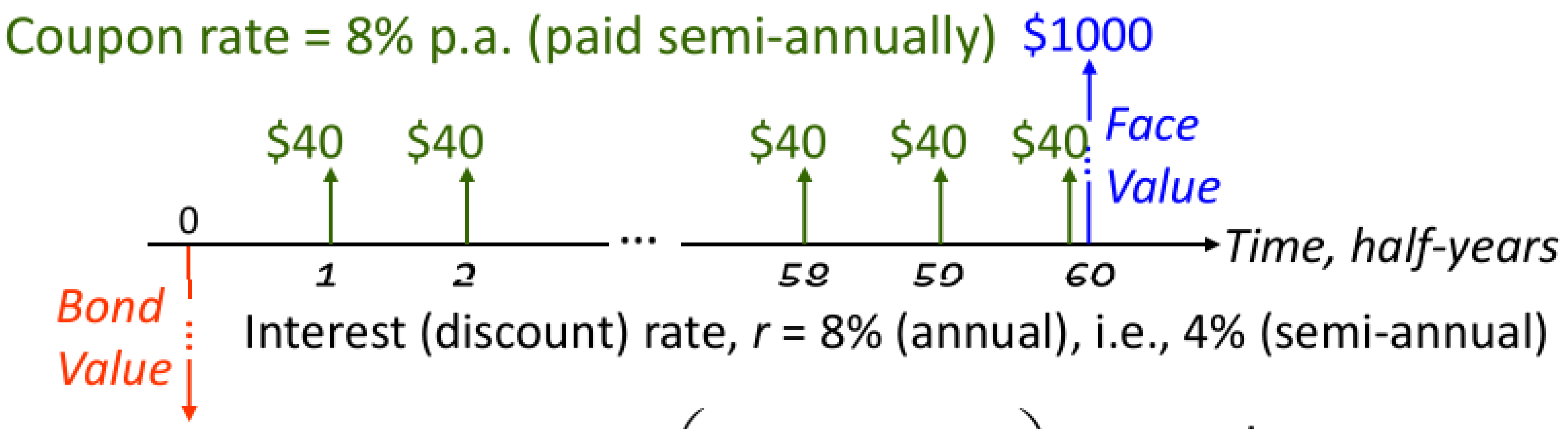
$$= \sum_{t=1}^{T} \frac{\textit{Coupon}}{(1+r)^t} + \frac{\textit{Face Value}}{(1+r)^T}$$

$$= \frac{\textit{Coupon}}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{\textit{Face Value}}{(1+r)^T}$$

$$\textit{Bond Value} = \textit{Coupon} \times \textit{Annuity Factor}(r, T) + \textit{Face Value} \times \textit{PV Factor}(r, T)$$

Example: Bond Priced at Par

- Assume a semi-annual discount rate of 4%, same as the semi-annual coupon rate

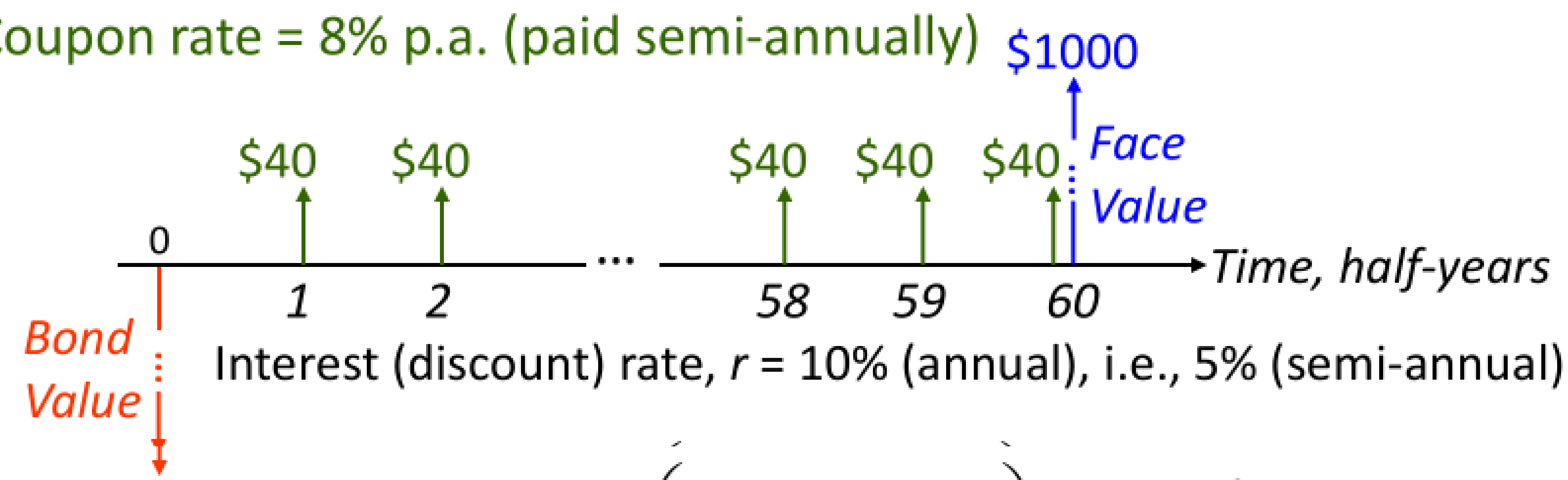


$$\$? = \frac{\$40}{0.04}\left(1 - \frac{1}{(1.04)^{60}}\right) + \frac{\$1000}{(1.04)^{60}}$$

$$\$1000 = \$904.94 + \$95.06$$

Example: Bond Priced at Discount

- Assume the semi-annual discount rate changes to 5%

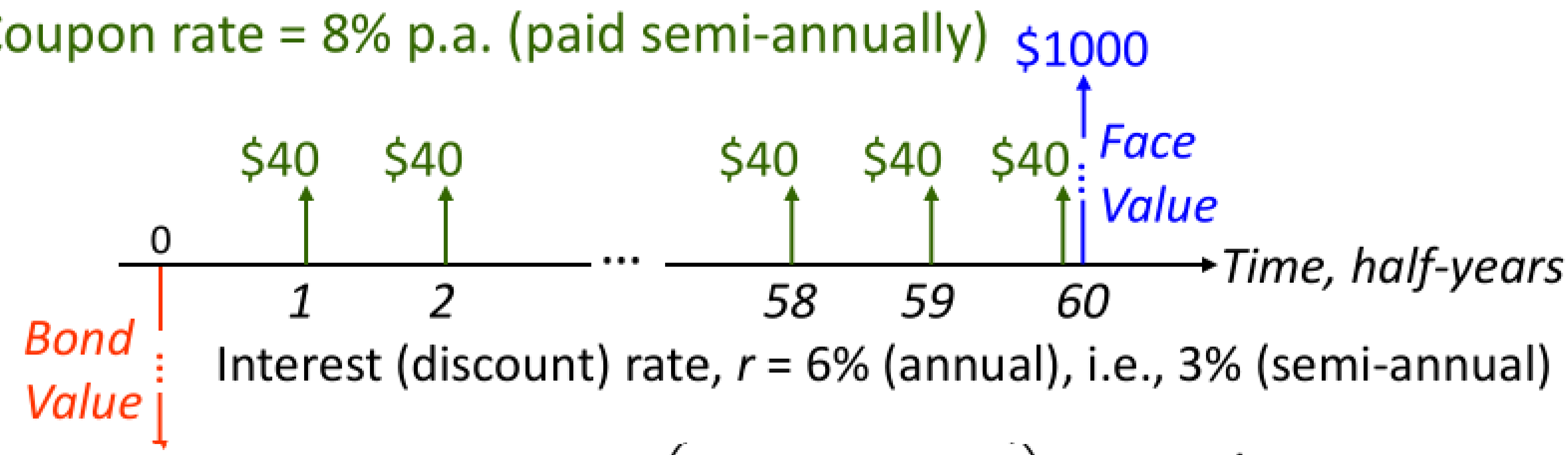


$$\$? = \frac{\$40}{0.05}\left(1 - \frac{1}{(1.05)^{60}}\right) + \frac{\$1000}{(1.05)^{60}}$$

$$\$810.71 = \$757.17 + \$53.54$$

Example: Bond Priced at Premium

- Assume the semi-annual discount rate changes to 3%

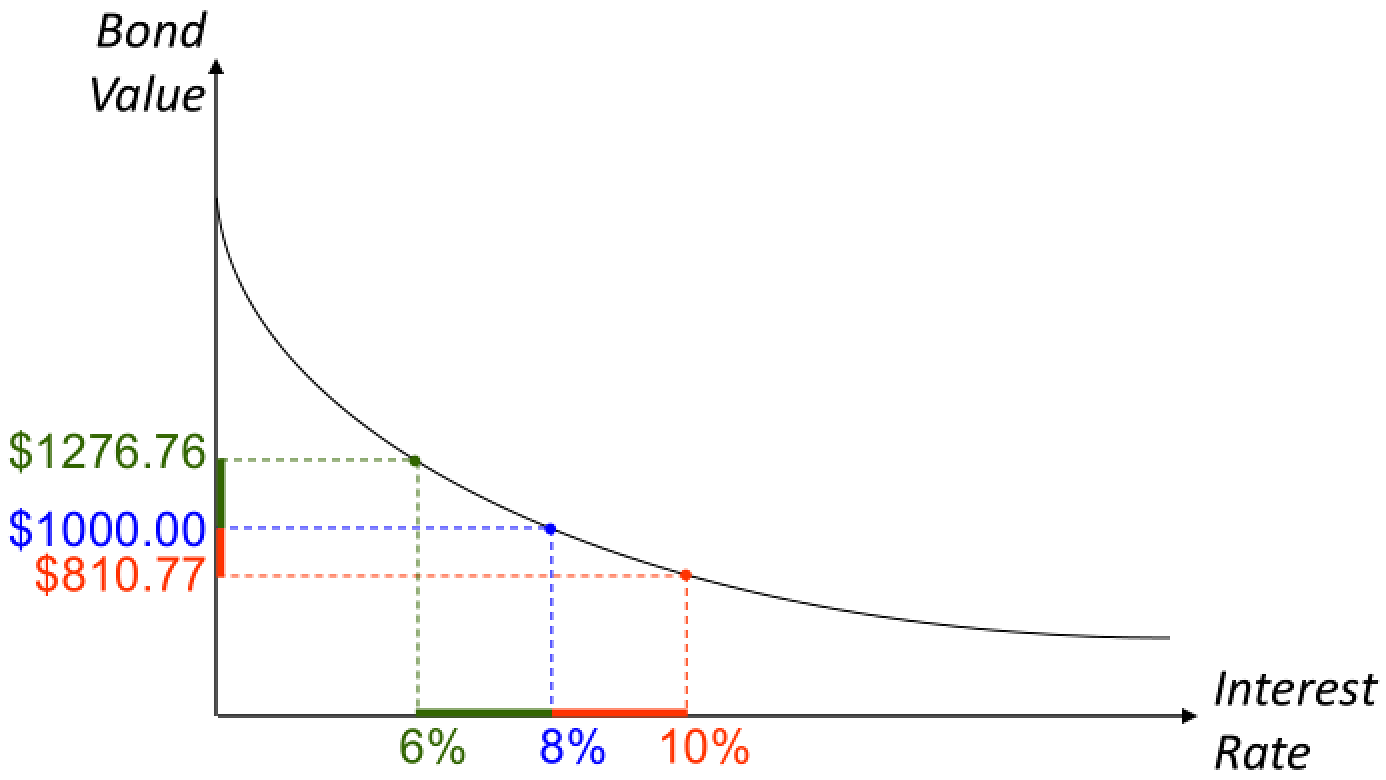


$$\$? = \frac{\$40}{0.03}\left(1-\frac{1}{(1.03)^{60}}\right)+\frac{\$1000}{(1.03)^{60}}$$

$$\$1276.76 = \$1107.02 + \$169.73$$

Bond Prices and Interest Rates

Bond price is an **inverse** and **convex** function of interest rates



Yield to Maturity

- Yield to Maturity: The interest rate that makes the PV of a bond equal to its price
  - The “internal rate of return” (IRR)
- Solve for *y* (we know everything else):

$$Bond\ Value = \frac{Coupon}{y}\left(1 - \frac{1}{(1+y)^T}\right) + \frac{Face\ Value}{(1+y)^T}$$

Problems with YTM

- YTM is the stated internal rate of return
  - It is not guaranteed to be the realized return due to interest rate risk
  - It may not be a correct description of expected return

- We want to take into account
  - Re-investment risk
  - Price risk
  - Default risk (for defaultable bonds)

Re-investment Risk

- YTM assumes that coupons are reinvested at the same yield

– As coupons are paid over time, the interest rate of debt instruments available for reinvestment is actually changing

- Consider the realized compound return (r) of a T-period investment

Initial Investment x (1 + r) T = Total Value at time T

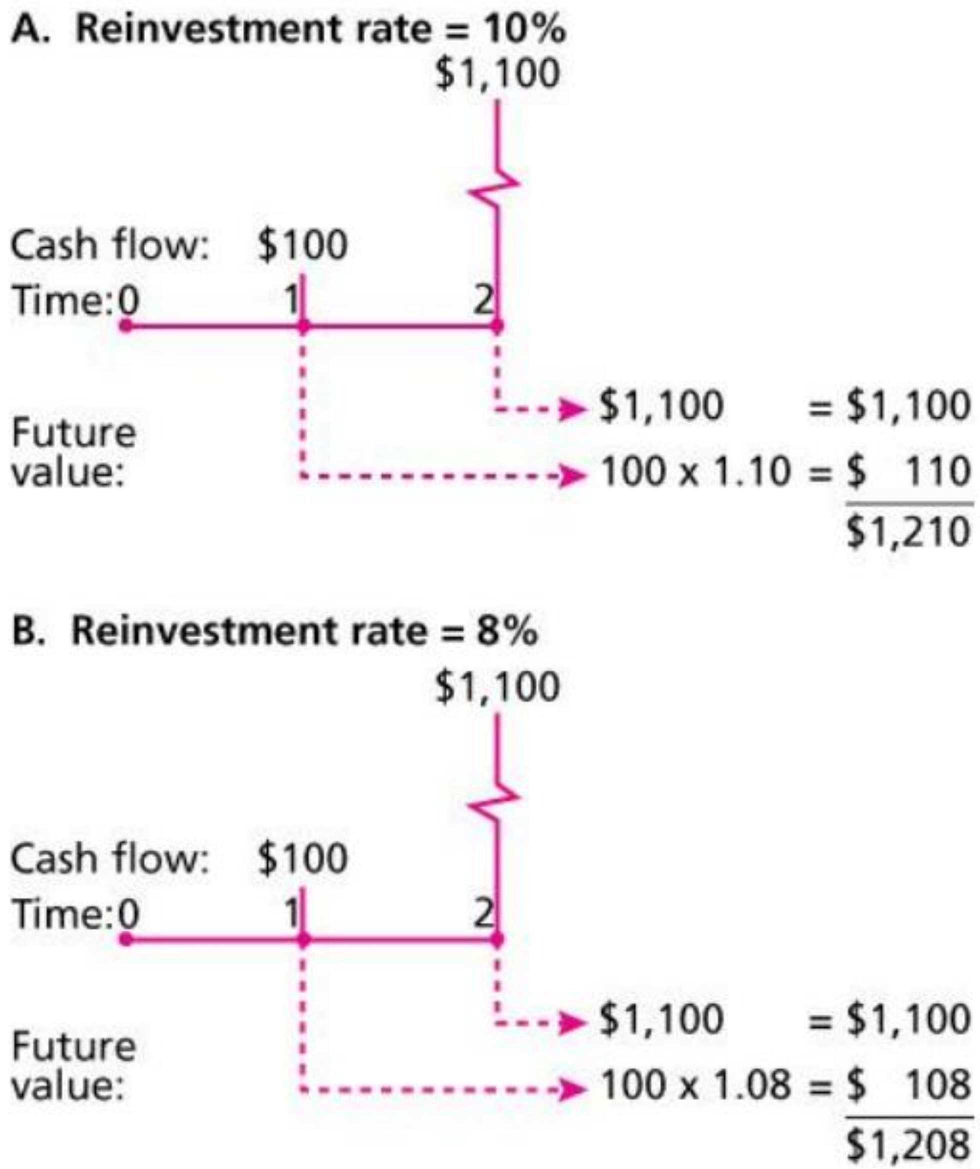
Note the total value is affected by re-investment income from interim payoffs and price changes of the security

- If the re-investment rate differs from YTM, then the realized compound return will be different

Re-investment Risk: Example

- A bond is paying an annual 10% coupon, has a face value of $1,000 and has 2 years to maturity; It is priced at par
- What is the YTM?
  - 10% (Why?)
- Calculate the realized compound return assuming the bond is held to maturity and coupons are reinvested at
  - 10%
  - 8%

Re-investment Risk: Example



Price Risk

- Price risk: the risk due to change of the sale price of a security

– If YTM is constant over time, return = YTM

– But, if YTM changes in the future, return will change as well

- Example: A 10% annual coupon bond with maturity of 2 years and a face value of $1,000 is selling at par

1. What is the YTM of this bond?
2. If the investor sells the bond in a year and the YTM remains constant, what is the return?
3. If the investor sells the bond in a year and the YTM changes to 8%, what is the return?

YTM vs. Realized Compound Return

- For coupon bonds, YTM = Realized Compound Return if:
  - Re-investment rate = YTM, and Investor holds bond to maturity
  - Re-investment rate = YTM, and YTM on sale date is the same as YTM on purchase date

- How about for zero-coupon bonds?

Default and Expected YTM: Example

- Casino Royale Co. issues bonds with 10 years to maturity, par value of $1,000, and a coupon rate of 9% with semi-annual payments; The bond is currently selling for $750

1. Calculate the stated YTM (What assumption about the bond’s cash flows is implicit in stated YTM?)

2. Suppose you expect Casino Royale will make good on all of the coupon payments but can only pay 70% of the face value at maturity. What is the expected YTM?

Yield Curve

- Term structure of interest rates: Relation between YTM and maturity

- Yield curve: a graph of the yields on Treasury bonds (y-axis) relative to the number of years to maturity (x-axis)

- Yield curve gives comparison of expected returns on short-term and long-term debt instruments

US Yield Curves

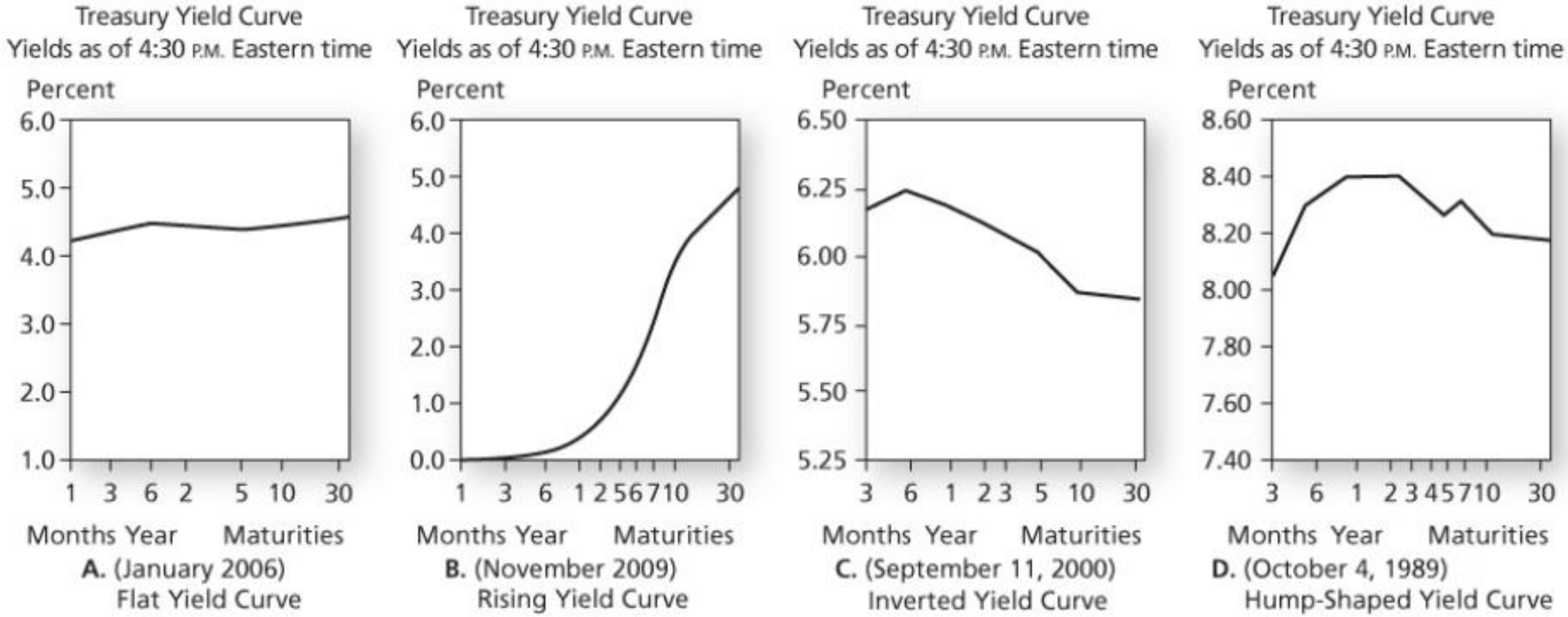
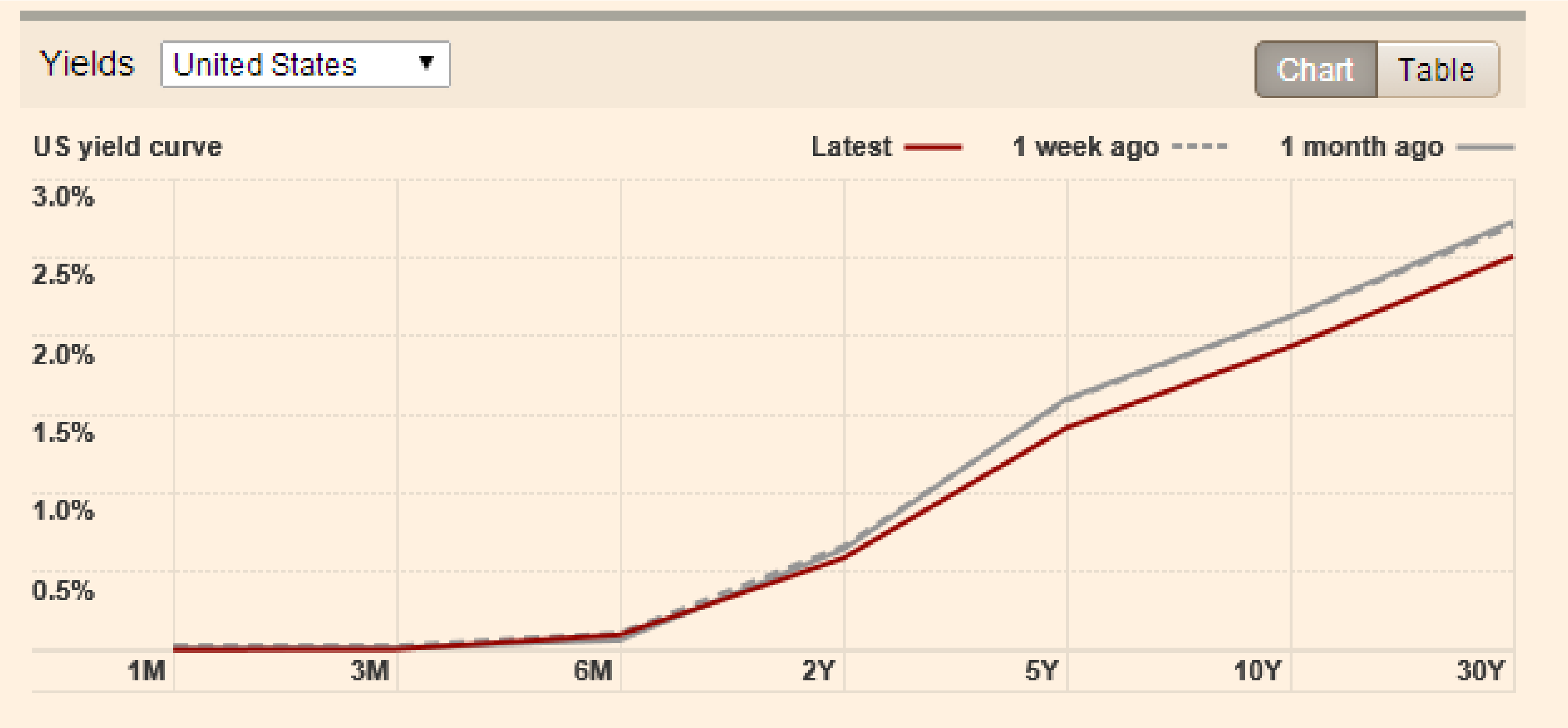


Figure 15.1 Treasury yield curves

Source: Various editions of *The Wall Street Journal*. Reprinted by permission of *The Wall Street Journal*, © 1989, 2000, 2006, and 2009 Dow Jones & Company, Inc. All Rights Reserved Worldwide.

US Yield Curve as of March. 23, 2015



Use Yield Curve to Price Bonds

- So far, we've assumed a constant interest rate

– Yields on different-maturity bonds are equal

– Each cash flow is discounted at the same interest rate, r

- In reality, interest rates are different

– Yields on different maturity bonds are not all equal

– Each cash flow must be considered as a stand-alone zero-coupon bond

$$\text{Bond Value} = PV(\text{Coupons}) + PV(\text{Face Value})$$

$$= \sum_{t=1}^{T} \frac{\text{Coupon}}{(1+y(t))^t} + \frac{\text{Face Value}}{(1+y(T))^T}$$

where y(t) is the yield to maturity of a t-year zero-coupon bond

- The interest rates depend on the time-to-maturity

Use Yield Curve to Price Bonds: Example

- Consider the following prices and yields to maturities on zero-coupon bonds ($1000 face value)

Maturity (years)	Yield to Maturity	Price
1	5%	$952.38 = $1,000/1.05
2	6%	$890.00 = $1,000/1.06^2
3	7%	$816.30 = $1,000/1.07^3

Find the price and YTM of a 3-year, 10% annual payment coupon Bond

$$Price = \frac{\$100}{1.05} + \frac{\$100}{1.06^2} + \frac{\$1,100}{1.07^3} = \$1,082.17$$

Solve for y in $\frac{\$100}{1+y} + \frac{\$100}{(1+y)^2} + \frac{\$1,100}{(1+y)^3} = \$1,082.17$

y = 6.88%

Term Structure of Interest Rates

- Theories which explain the shape of the yield curve
  - Pure Expectations Theory
  - Liquidity Premium Theory
  - Segmented Markets Theory

Theories of The Term Structure
Expectations Hypothesis

- Yield curve slope reflects market expectations of future interest rates
  - Assumes investor has no maturity preferences and transaction costs are low
  - Implied forward rate is the market's forecast of the future interest rate

- Upward (downward) yield curve means that the market is expecting higher (lower) future short term rates
- Flat yield curve means that short rate is expected to stay at the same level

Implied Forward Rates

- By the expectations hypothesis, use the term structure of interest rates below and find the implied forward rates:

- E.g.
  - 1-year Treasury bill 1% p.a. (per annum)
  - 2-year Treasury note 2% p.a.
  - 3-year Treasury note 3% p.a.
    - What’s the one-year forward rate from year 1 to 2
    - What’s the two-year forward rate from year 1 to 3

Theories of The Term Structure
Liquidity Preference

- Investors prefer to hold short-term bonds (because the return on longer-term bonds are uncertain)
  - They require a risk premium to hold longer-term bonds
  - This liquidity premium compensates short-term investors for the uncertainty about future prices

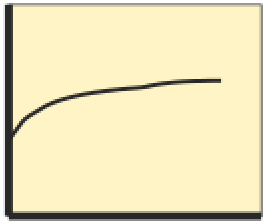
Interpreting the Term Structure

- The yield curve reflects expectations of future interest rates
- The forecasts of future rates are clouded by other factors, such as liquidity premiums

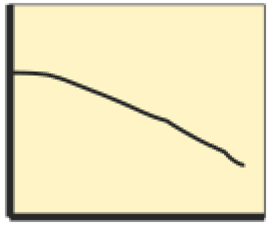
- An upward sloping curve could indicate:
  - Rates are expected to rise, and/or
  - Investors require large liquidity premiums to hold long term bonds
- The yield curve is a good predictor of the business cycle
  - Long term rates tend to rise in anticipation of economic expansion
  - Inverted yield curve may indicate that interest rates are expected to fall and signal a recession

Predicting recessions

- Expected higher interest rate levels in the future.
- Interest rates low now means we are in expansive monetary policy. Low interest rates to stimulate growth.
- Expanding economy ahead

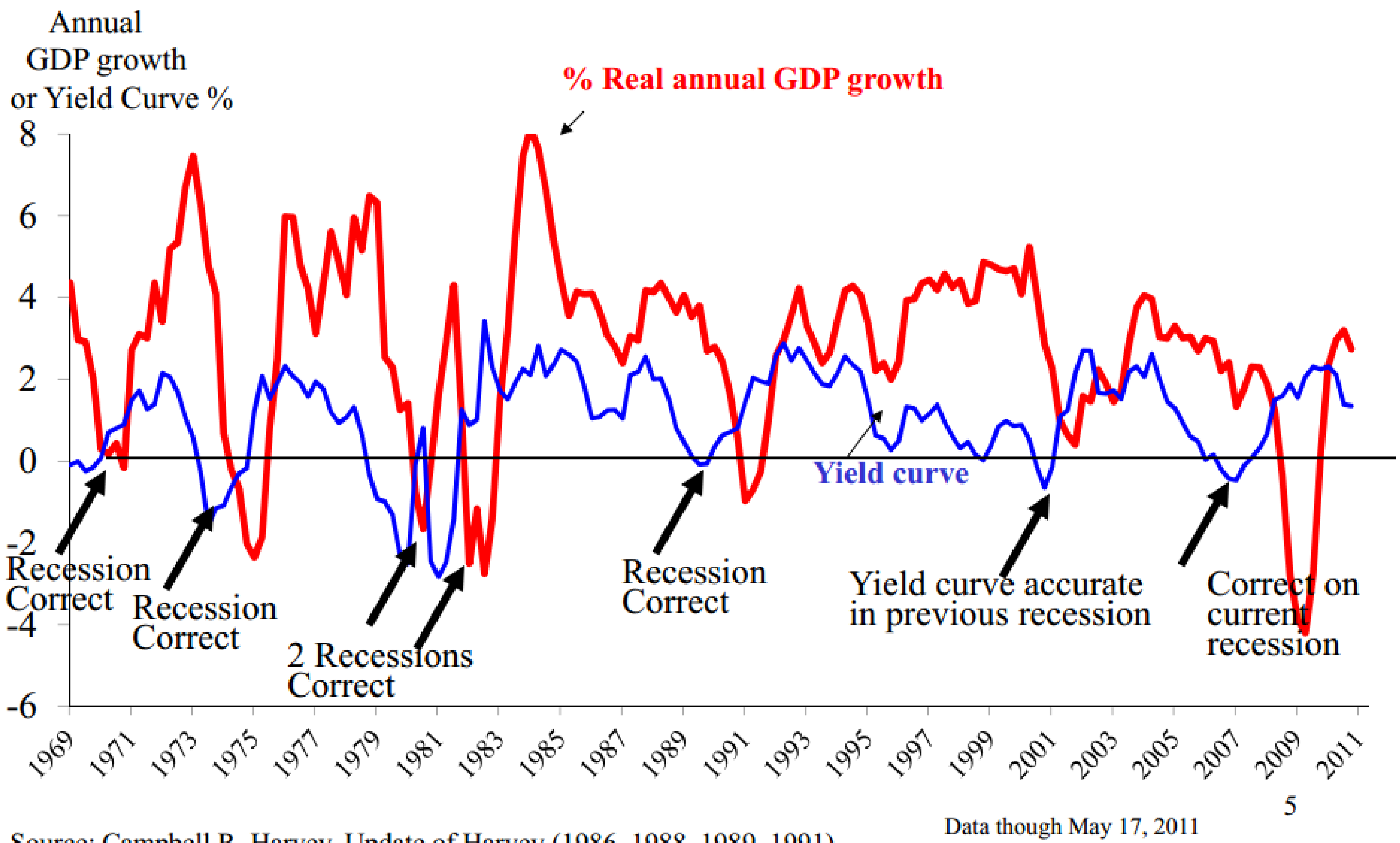


- Expected lower interest rate levels in the future
- Tight monetary policy: High interest rates now to prevent economy from overheating.
- Investors demand longer term bonds so much that long term yields are low. Willingness to lock in low rates could mean few investment opportunities in economy.
- Recession coming soon?



Yield Curve Inverts Before Last Seven Recessions

(5-year Treasury note minus 3-month Treasury bill yield – constant maturity)



Source: Campbell R. Harvey. Update of Harvey (1986, 1988, 1989, 1991).

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http://www.duke.edu/~charvey/research_term_structure.htm

Bond market 2
-Markets and Institutions

Ruichang LU (卢瑞昌)

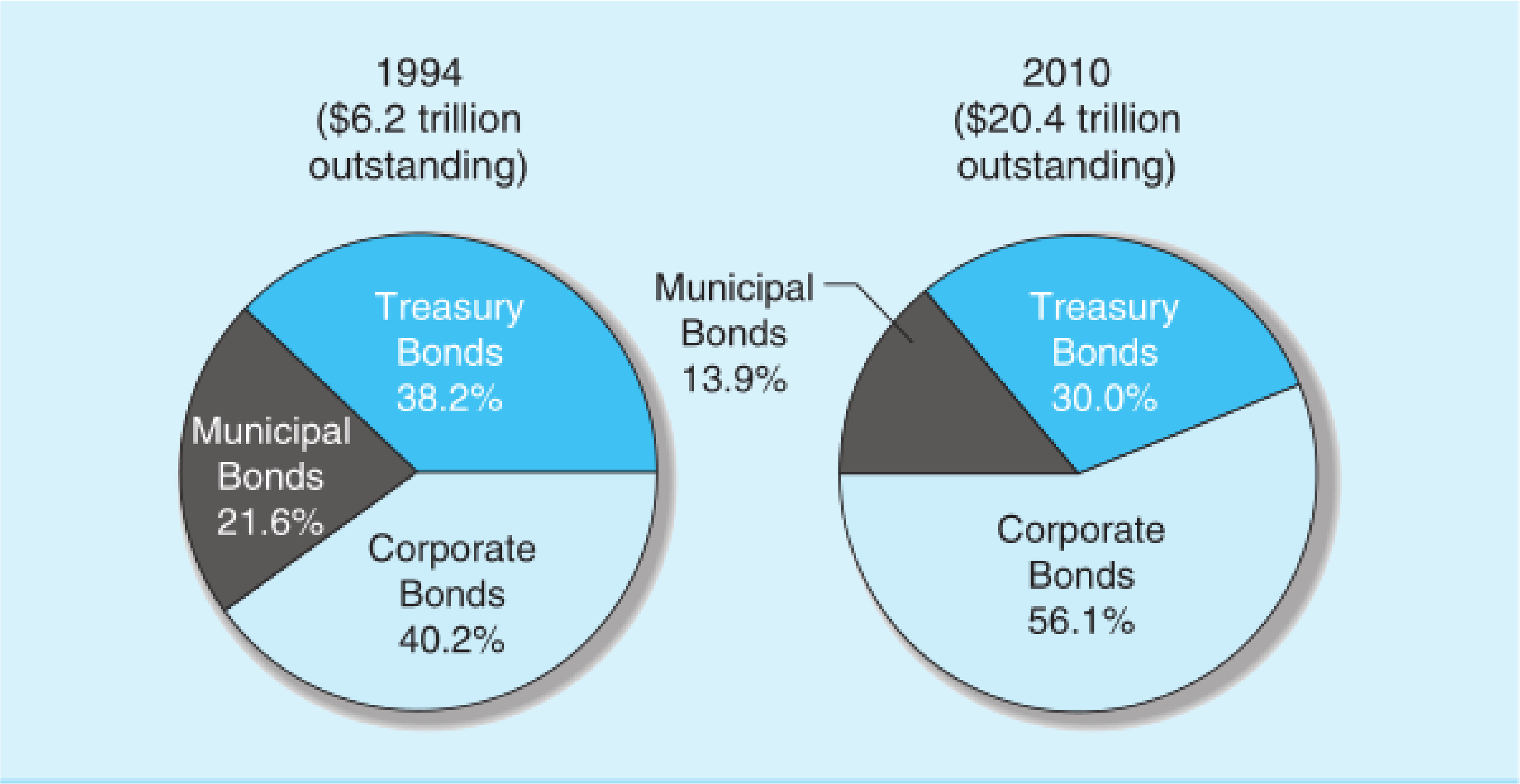
Department of Finance
Guanghua School of Management
Peking University

Outline

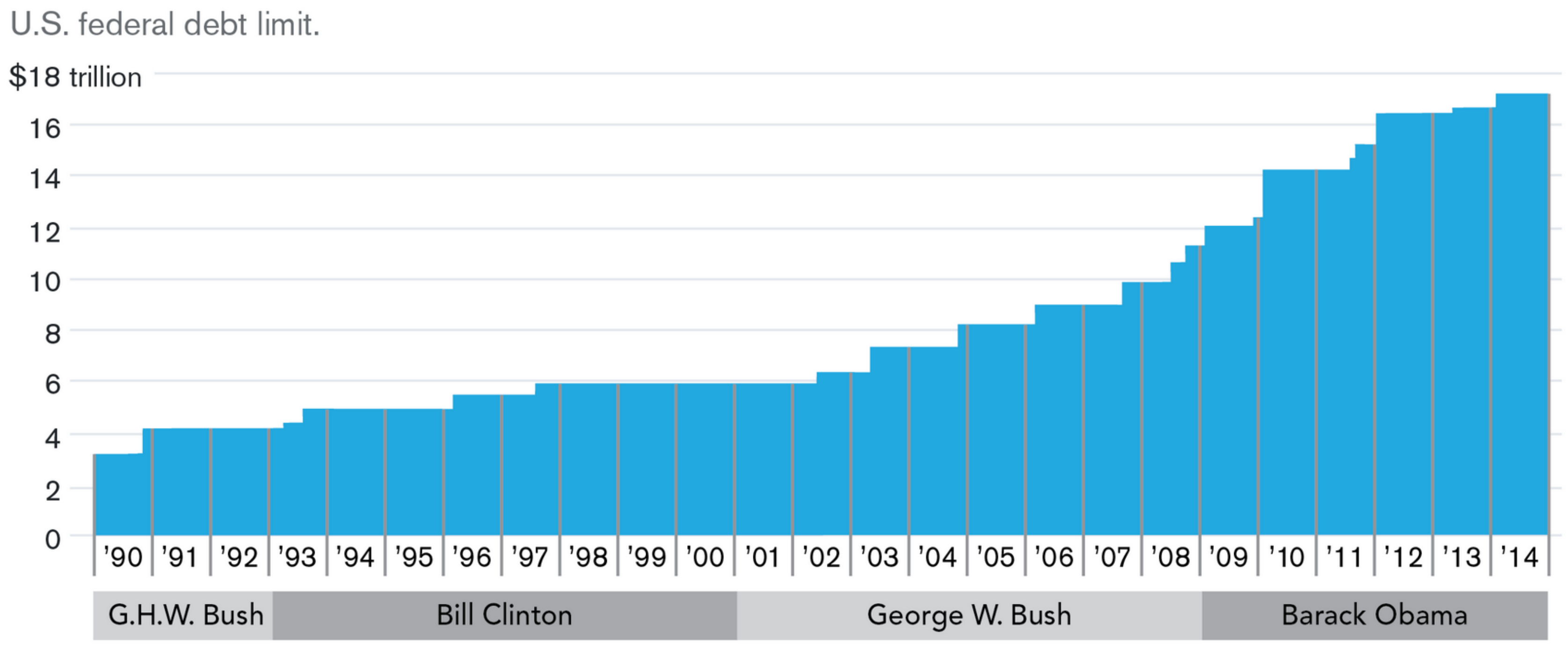
- Treasury notes and bonds
- Municipal bonds
- Corporate bonds
- International bond market
- Advanced concept in bond market
  - Duration
  - Convexity

Bond Market Instruments Outstanding

Figure 6–1 Bond Market Instruments Outstanding, 1994–2010

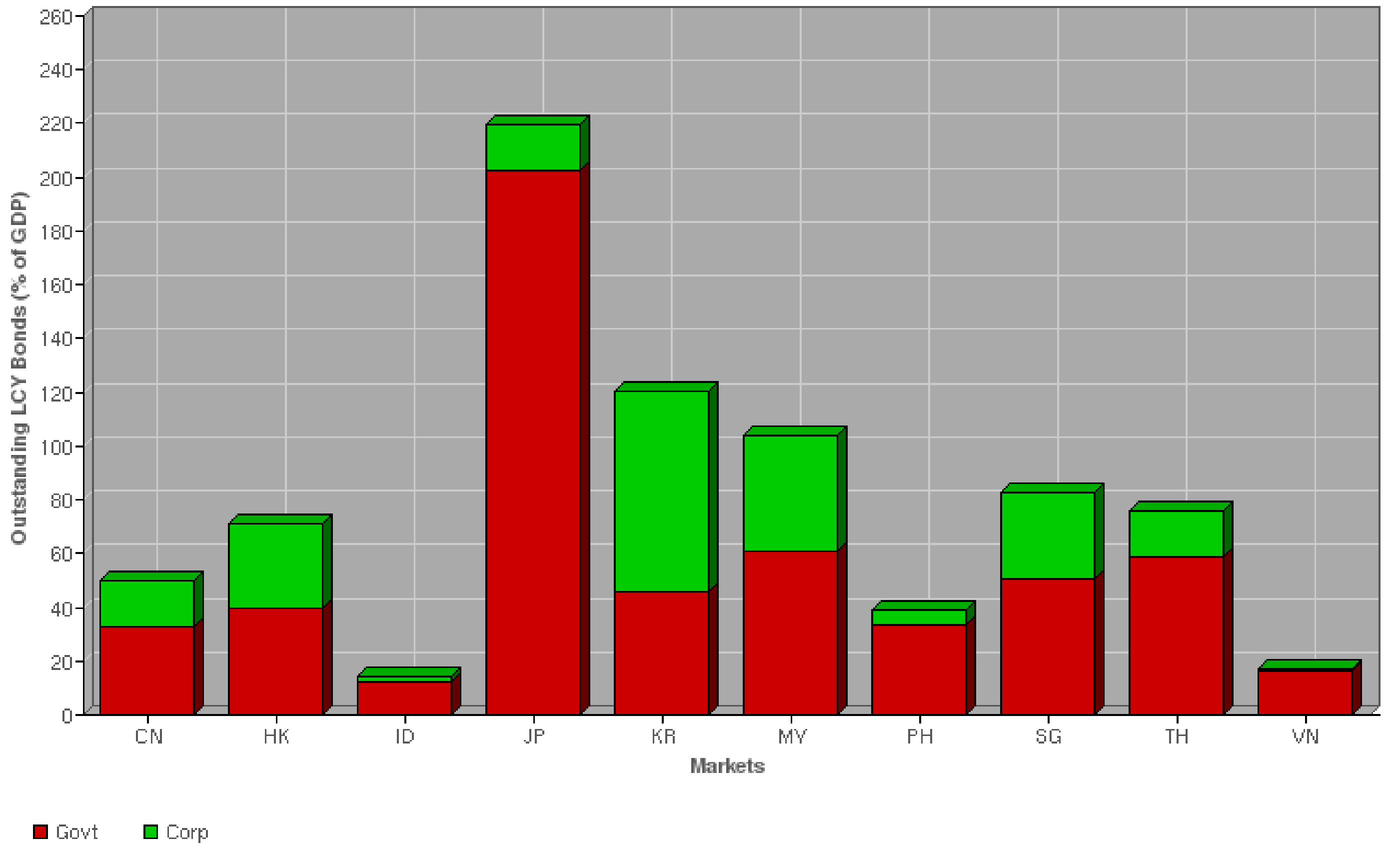


US debt ceiling



http://www.bloombergview.com/quicktake/the-debt-ceiling

Debt/GDP



Country name	2010	2011	2012	2013
United States	85.6	90.1	94.3	96.1
Japan	174.8	189.5	196.0	

Size of the market

- Global Bond volume totaled \$6.29tr in 2014.

- According to the Federal Reserve Bank of New York, the average daily trading volume in T-note and T-bond issues for the week ended July 7, 2010 was \$427.74 billion.

- Source: Dealogic.com

T-notes and T-bonds

- T-notes and T-bonds issued by the U.S. Treasury to finance the national debt and other federal government expenditures
- Backed by the full faith and credit of the government and are default risk free
- Pay relatively low rates of interest (yields to maturity)
- Given their longer maturity, not entirely risk free due to interest rate fluctuations
- Pay coupon interest (semiannually): notes have maturities from 1-10 years; bonds 10-30 years

Types of issue

- The Treasury issues two types of notes and bonds: fixed principal and inflation-indexed.
  - While both types pay interest twice a year, the principal value used to determine the percentage interest payment (coupon) on inflation-indexed bonds is adjusted to reflect inflation (measured by the consumer price index). Thus, the semiannual coupon payments and the final principal payment are based on the inflation-adjusted principal value of the security.

Types of issue

- For example, a two-year, 10 percent coupon (annual) bond issued with a principal value (face value) of $1,000 will pay a total of $10 and $10 in the first and second years.
- An indexed (annual) bond when inflation is 10 percent in the first year and the second year will pay a 10 percent coupon based on principal values of $1,000 X(1.1) =$1,100 and $1,000X(1.1)X1.1= $1,210, respectively.
  - That is, the first year coupon will be 10% $1,100 $11 and the second year coupon will be 10% $1,210 $12.10.

Separate Trading of Registered Interest and Principal Securities (STRIPS).

- A treasury security in which the individual interest payments are separated from the principal payment
- Effectively creates sets of securities--one for each semiannual interest payment one one for the final principal payment
- Often referred to as "Treasury zero-coupon bonds"
- Created by U.S. Treasury in response to separate trading of treasury security principal and interest developed by securities firms; only available through FIs and government securities brokers

Creation of STRIP

TABLE 6–2 Present Value of STRIP Components of a 5-Year T-Note with an 8 Percent Coupon Rate and 7.90 Percent Yield to Maturity

Maturity on Security (in years)	Cash Flow Received at Maturity	Present Value of Cash Flow at 7.90 Percent
0.5	$ 400	$ 384.80
1.0	400	370.18
1.5	400	356.11
2.0	400	342.58
2.5	400	329.56
3.0	400	317.04
3.5	400	304.99
4.0	400	293.40
4.5	400	282.25
5.0	400	271.53
5.0	10,000	6,788.21
Total		$10,040.65

Usage of STRIP

- **Using a STRIP to Immunize against Interest Rate Risk**

- Investors needing a lump sum payment in the distant future (e.g., life insurers) would prefer to hold the principal portion of the STRIP.
- Investors wanting nearer-term cash flows (e.g., commercial banks) would prefer the interest portions of the STRIP.
- Also, some state lotteries invest the present value of large lottery prizes in STRIPs to be sure that funds are available to meet required annual payments to lottery winners.
- Pension funds purchase STRIPs to match payment cash flows received on their assets (STRIPs) with those required on their liabilities (pension contract payments).

Auction in T-bond market

- Similar to the primary market T-bill sales, the Treasury sells T-notes and bonds through competitive and noncompetitive auctions

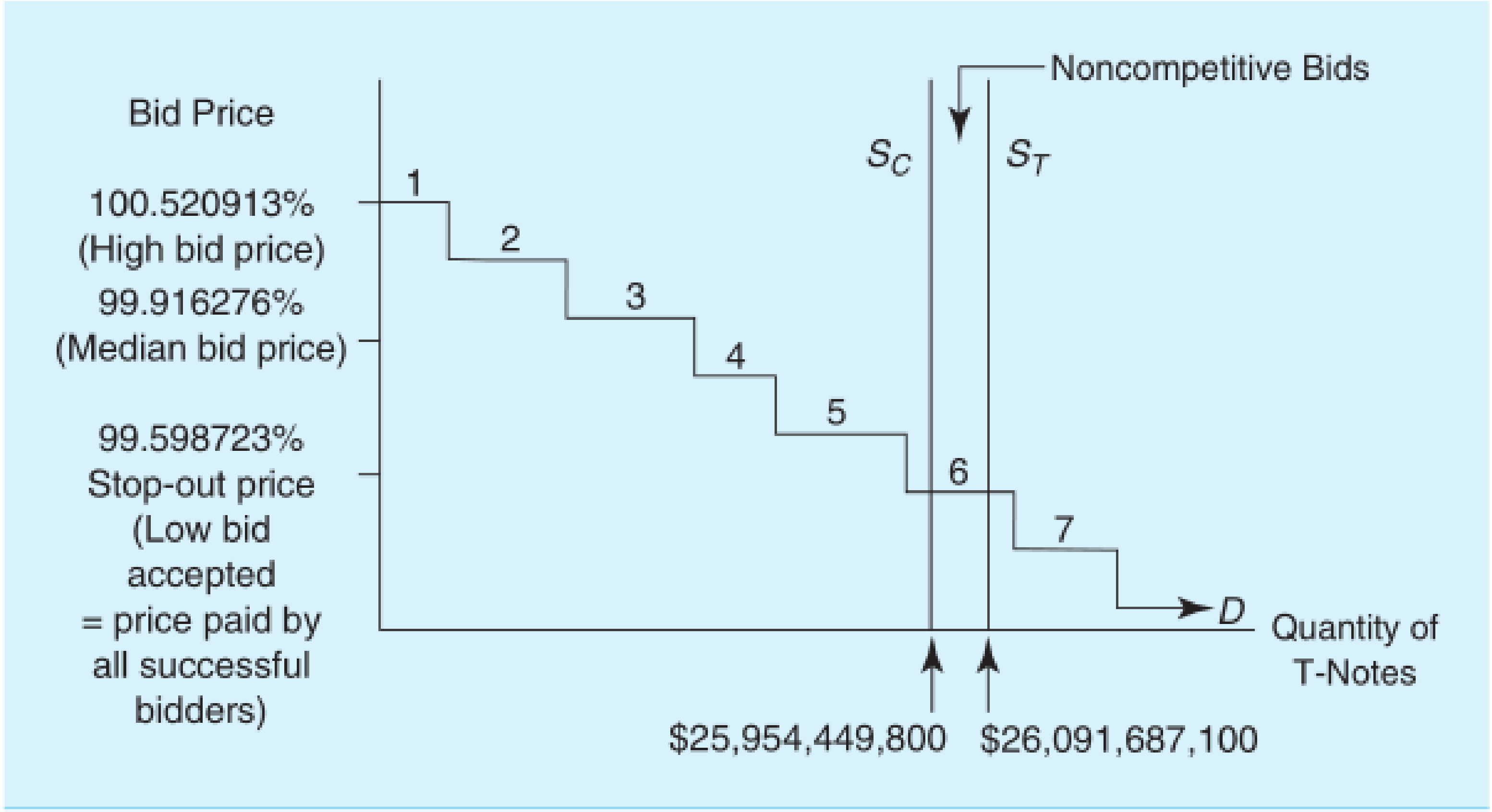
- **Auction Pattern for Treasury Notes and bonds**

Security	**Purchase Minimum**	**General Auction Schedule**
2-year note	$1,000	Monthly
5-year note	$1,000	Feb, May-Aug, Nov
10-year note	$1,000	Feb, May-Aug, Nov

- Most government bond auctions are conducted as uniform price auctions.
- Investors submit bids expressed in terms of % p.a. yield.
- Uniform payment at the price determined at the cut-off bid.
- For competitive auctions, securities are allotted from the lowest to the highest bidder.
  - At the highest cutoff bid, the allotment is on a pro-rated basis.

http://www.treasurydirect.gov/instit/annceresult/press/preanre/2015/A_20150305_5.pdf

Figure 6–7 **Treasury Auction Results**



Municipal bonds

- **Securities issued by state and local governments**
- **Tax receipts or revenues generated are the source of repayment**
- **Attractive to household investors because interest (but not capital gains) are tax exempt**
  - (in contrast, interest payments on Treasury securities are exempt only from state and local income taxes)
  - As a result, the interest borrowing cost to the state or local government is lower, because investors are willing to accept lower interest rates on municipal bonds relative to comparable taxable bonds such as corporate bonds.

Conversion of a Municipal Bond Rate to a Tax Equivalent Rate

- **$i_a = i_b(1 - t)$**

- Where:
- i_a = After-tax (equivalent tax exempt) rate of return on a taxable
- bond
- i_b = Before-tax rate of return on a taxable bond
- t = Income tax rate of the marginal bond holder

- **Example:** You can invest in taxable corporate bonds that are paying
- 10% annually on munis. Your marginal tax rate is 28%. The after-
- tax rate of return on the taxable bond is:

- 10%(1-.28) = 7.2%

Primary Market Placement Choices for Munis

- **General Public Offering**
  - underwriter is selected either by negotiation or by competitive bidding
  - the underwriter offers the bonds to the general public
- **Rule 144A Placement**
  - bonds are sold on a semi-private basis to qualified investors (generally Fis, Qualified Institutional Buyers)
  - Secondary market trading is thin. Why?

Corporate Bonds

- **All long-term bonds issued by corporations**
- **Minimum denominations publicly traded corporate bonds is $1,000**
- **Generally pay interest semiannually**
- **Bond indenture**

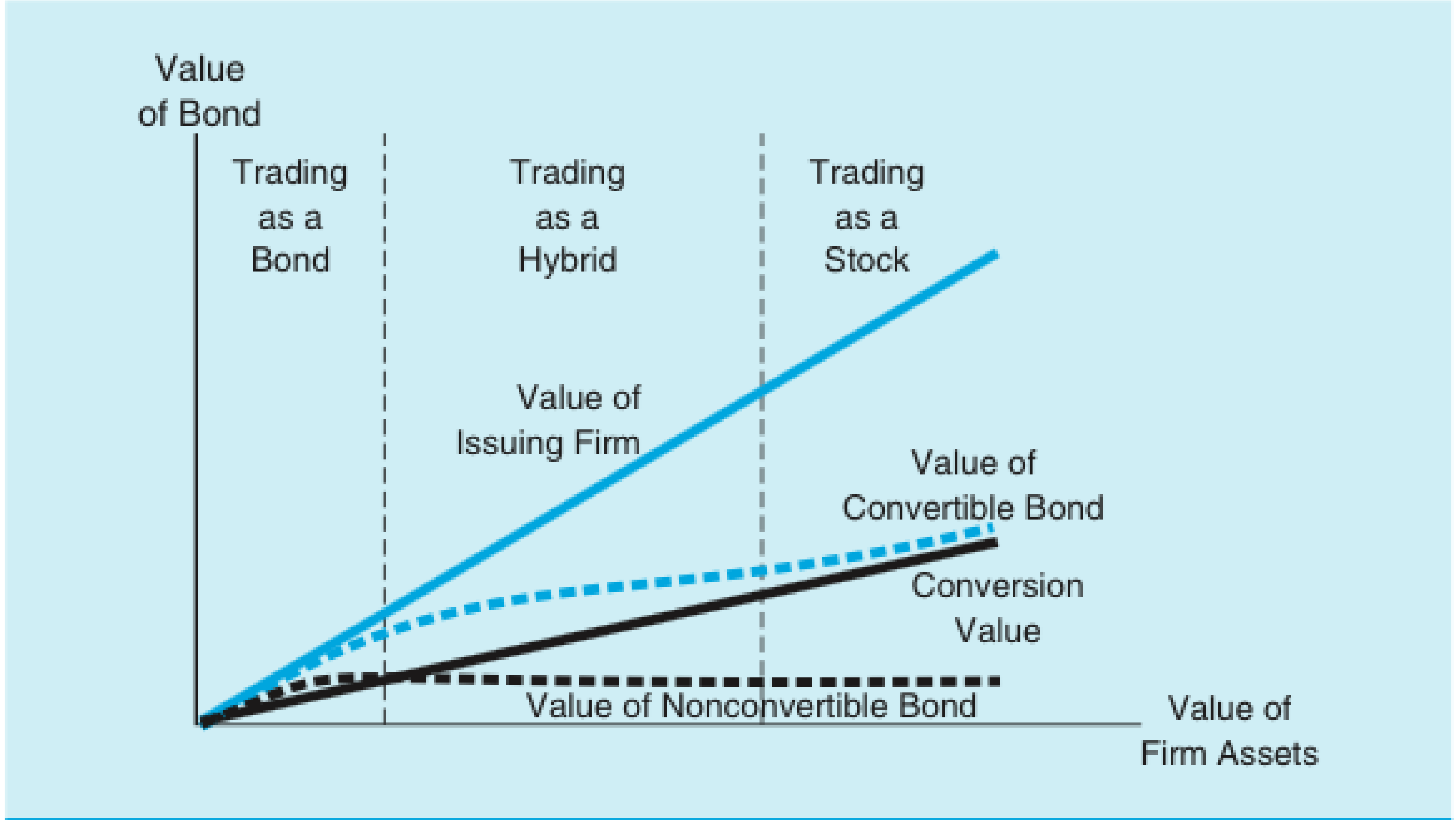
Indenture

- The **bond indenture** is the legal contract that specifies the rights and obligations of the bond issuer and the bond holders.
- The bond indenture contains a number of covenants associated with a bond issue. These bond covenants describe rules and restrictions placed on the bond issuer and bond holders.
    - the ability to call the bond issue
    - restrictions as to limits on the ability of the issuer to increase dividends paid to equity holders.
    - the bond indenture helps lower the risk (and therefore the interest cost) of the bond issue.

Convertible bonds

- **Convertible bonds** are bonds that may be exchanged for another security of the issuing firm (e.g., common stock) at the discretion of the bond holder. If the market value of the securities the bond holder receives with conversion exceeds the market value of the bond, the bond holder can return the bonds to the issuer in exchange for the new securities and make a profit.
- An example of a convertible bond issue is the bond issue by General Motors Corporation in 2003. These bonds pay an annual coupon of 6.25 percent. Furthermore, on their maturity date, the holder of the bond has the option to choose between receiving the nominal value in cash or converting the bond into 21 shares of the General Motors Corporation stock.

Figure 6–9 **Value of a Convertible Bond**



Bond Ratings

	Moody' s	Standard & Poor's	Safety
Investment Grade	Aaa	AAA	The strongest rating; ability to repay interest and principal is very strong.
	Aa	AA	Very strong likelihood that interest and principal will be repaid
	A	A	Strong ability to repay, but some vulnerability to changes in circumstances
	Baa	BBB	Adequate capacity to repay; more vulnerability to changes in economic circumstances
Speculative	Ba	BB	Considerable uncertainty about ability to repay.
	B	B	Likelihood of interest and principal payments over sustained periods is questionable.
	Caa	CCC	Bonds in the Caa/CCC and Ca/CC classes may already be
	Ca	CC	in default or in danger of imminent default
	C	C	C-rated bonds offer little prospect for interest or principal on the debt ever to be repaid.

Bond spread

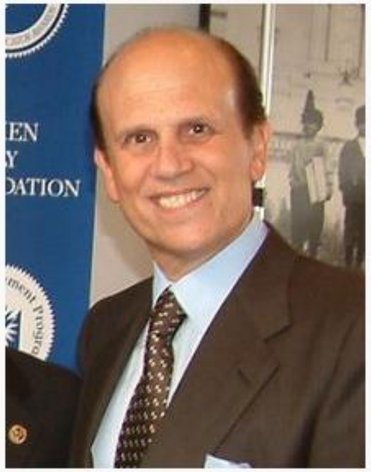
REUTERS CORPORATE BOND SPREADS

Rating	1 yr	2 yr	3 yr	5 yr	7 yr	10 yr	30 yr
Aaa/AAA	14	16	27	40	56	68	90
Aa1/AA+	22	30	31	48	64	77	99
Aa2/AA	24	37	39	54	67	80	103
Aa3/AA-	25	39	40	58	71	81	109
A1/A+	43	48	52	65	79	93	117
A2/A	46	51	54	67	81	95	121
A3/A-	50	54	57	72	84	98	124
Baa1/BBB+	62	72	80	92	121	141	170
Baa2/BBB	65	80	88	97	128	151	177
Baa3/BBB-	72	85	90	102	134	159	183
Ba1/BB+	185	195	205	215	235	255	275
Ba2/BB	195	205	215	225	245	265	285
Ba3/BB-	205	215	225	235	255	275	295
B1/B+	265	275	285	315	355	395	445
B2/B	275	285	295	325	365	405	455
B3/B-	285	295	305	335	375	415	465
Caa/CCC+	450	460	470	495	505	515	545
US Treasury Yield	4.74	4.71	4.68	4.63	4.60	4.59	4.56

Spread values represent basis points (bps) over a US Treasury security of the same maturity, or the closest matching maturity.

Junk bond market history

Michael Milken



Born	Michael Robert Milken July 4, 1946 (age 68) Encino, California, U.S.
Citizenship	United States
Alma mater	University Of California, Berkeley The Wharton School
Occupation	Businessman, financier, philanthropist
Known for	developing the High-yield bond market, Indictment for securities fraud, philanthropy

In 1980s, Junk bond issuance 170 Billion, Drexel Burnham Lambert underwrote 80 Billion.

International Aspects of Bond Markets

- **International bond market**
  - trades bonds that are underwritten by an international syndicate
  - offer bonds simultaneously to investors in several countries
  - issue bonds outside the jurisdiction of any single country
  - offer bonds in unregistered form

Eurobonds, Foreign Bonds, Brady Bonds and Sovereign Bonds

- **Eurobonds**
- **Foreign Bonds**
- **Sovereign Bonds**

Eurobonds

- Eurobonds are long-term bonds issued and sold outside the country of the currency in which they are denominated (e.g., dollar-denominated bonds issued in Europe or Asia).
- Perhaps confusingly, the term *Euro* simply implies the bond is issued outside the country in whose currency the bond is denominated. Thus, “Euro”-bonds are issued in countries outside of Europe and in currencies other than the euro. Indeed, the majority of issues are still in U.S. dollars and can be issued in virtually any region of the world.

Eurobonds

- Eurobonds were first sold in 1963 as a way to avoid taxes and regulation. U.S. corporations were limited by regulations on the amount of funds they could borrow domestically (in the United States) to finance overseas operations, while foreign issues in the United States were subject to a special 30 percent tax on their coupon interest. In 1963, these corpora- tions created the Eurobond, by which bonds were denominated in various currencies and were not directly subject to U.S. regulation. Even when these regulations were abandoned, access to a new and less-regulated market by investors and corporations created sufficient demand and supply for the market to continue to grow.

Foreign Bonds

- **Foreign Bonds.** Foreign bonds are long-term bonds issued by firms and governments outside of the issuer's home country and are usually denominated in the currency of the country in which they are issued rather than in their own domestic currency—for example, a Japanese company issuing a dollar-denominated public bond rather than a yen-denominated bond in the United States.

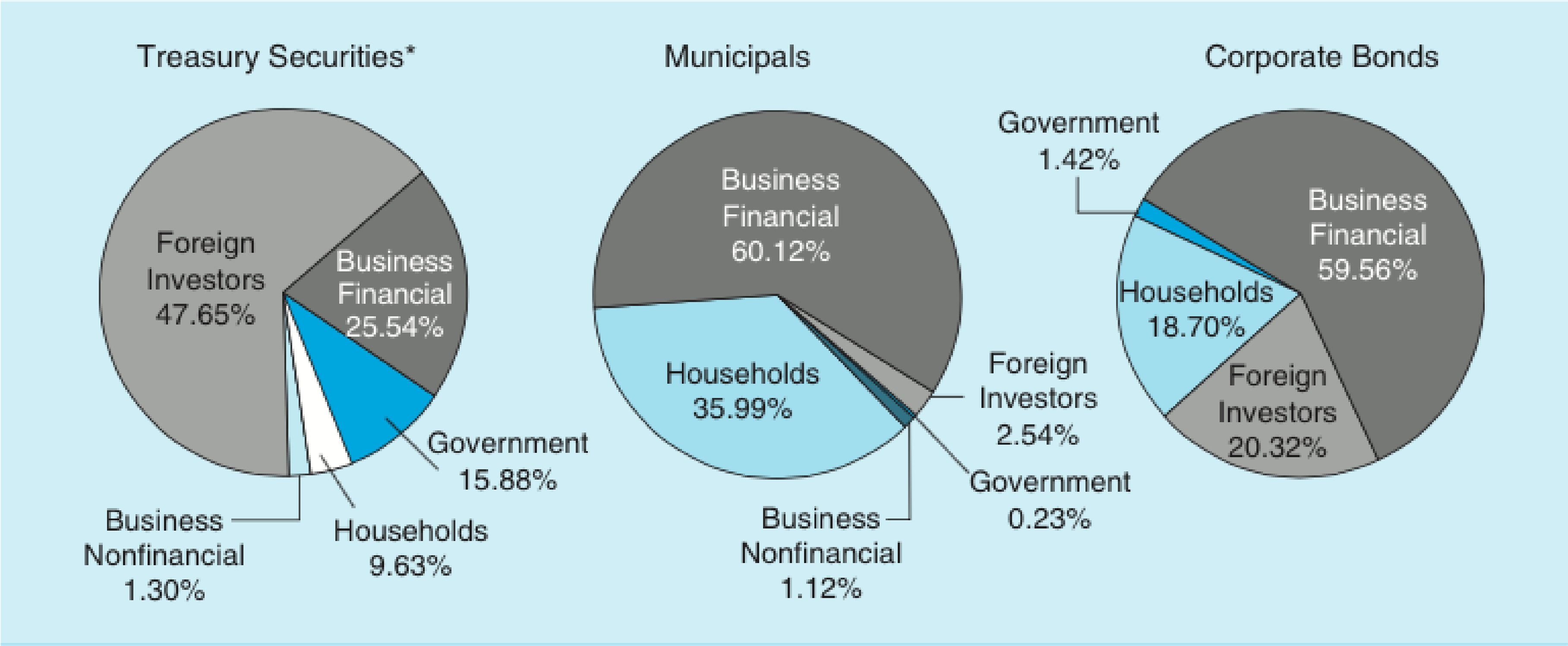
Sovereign bonds

- **Sovereign bonds** are government-issued debt. Sovereign bonds have historically been issued in foreign currencies, in either U.S. dollars or euros.

- http://www.bis.org/publ/qtrpdf/r_qa1503.pdf

Institutions

Figure 6–11 **Bond Market Securities Held by Various Groups of Market Participants, 2010**



Reading Quotes

U.S. Treasury Quotes

TREASURY NOTES & BONDS

GO TO: Bills

Friday, March 27, 2015 Find Historical Data | WHAT'S THIS?

Treasury note and bond data are representative over-the-counter quotations as of 3pm Eastern time. For notes and bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

Maturity	Coupon	Bid	Asked	Chg	Asked yield
8/15/2015	4.250	101.5859	101.6016	-0.0078	0.048
8/15/2015	10.625	103.9766	103.9922	-0.0625	0.145
8/31/2015	0.375	100.1016	100.1172	0.0156	0.095
8/31/2015	1.250	100.4688	100.4844	0.0078	0.092
9/15/2015	0.250	100.0625	100.0781	0.0391	0.080
9/30/2015	0.250	100.0469	100.0625	0.0234	0.126
9/30/2015	1.250	100.5547	100.5703	0.0156	0.115
10/15/2015	0.250	100.0391	100.0547	0.0156	0.149
10/31/2015	0.250	100.0391	100.0547	0.0313	0.157
10/31/2015	1.250	100.6172	100.6328	0.0156	0.168
11/15/2015	0.375	100.1094	100.1250	0.0234	0.175
11/15/2015	4.500	102.6953	102.7109	unch.	0.172
11/15/2015	9.875	106.0469	106.0625	-0.0391	0.196
11/30/2015	0.250	100.0234	100.0391	0.0156	0.192
11/30/2015	1.375	100.7891	100.8047	0.0313	0.173
12/15/2015	0.250	100.0391	100.0547	0.0391	0.173

Monday, December 31, 2007

Treasury Quotes

U.S. Government Bonds and Notes

Representative Over-the-Counter quotation based on transactions of $1 million or more. Treasury bond, note and bill quotes are from midafternoon. Colons in bond and note bid-and-asked quotes represent 32nds; 101:01 means 101 1/32. Net change in 32nds. n-Treasury Note. i-Inflation-indexed issue. Treasury bill quotes in hundredths, quoted in terms of a rate of discount. Days to maturity calculated from settlement date. All yields are to maturity and based on the asked quote. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues quoted below par.
*-When issued. Daily change expressed in basis points.

Rate	Maturity Mo/Yr	Bid	Asked	Chg	Asked Yield
4 3/8	Jan 08 n	100:02	100:03	-1	2.84
3	Feb 08 n	100:00	100:00	....	2.91
5 1/2	Feb 08 n	100:08	100:09	-1	2.91
3 3/8	Feb 08 n	100:00	100:01	-1	2.91
4 5/8	Feb 08 n	100:06	100:07	-1	3.02
4 5/8	Mar 08 n	100:10	100:11	....	3.14
4 7/8	Apr 08 n	100:15	100:16	....	3.23

Example 6-4 Calculation of a T-Note Price from a *Wall Street Journal Online* Quote

In Table 6–1, look at the T-note outstanding on Friday July 16, 2010 (with a settlement date of Monday, July 19, 2010), with a maturity on November 15, 2013 (i.e., they were 3.3260274 years from maturity). The T-note had a coupon rate of 4.250 percent and an asked yield of 1.0030 percent. Using the bond valuation formula, the asked price on the bond should have been:

$$V_b = \frac{4.250}{2}\left[\frac{1 - \frac{1}{(1 + .01003/2)^{2(3.3260274)}}}{0.01003/2}\right] + 100\left[\frac{1}{(1 + .01003/2)^{2(3.3260274)}}\right]$$

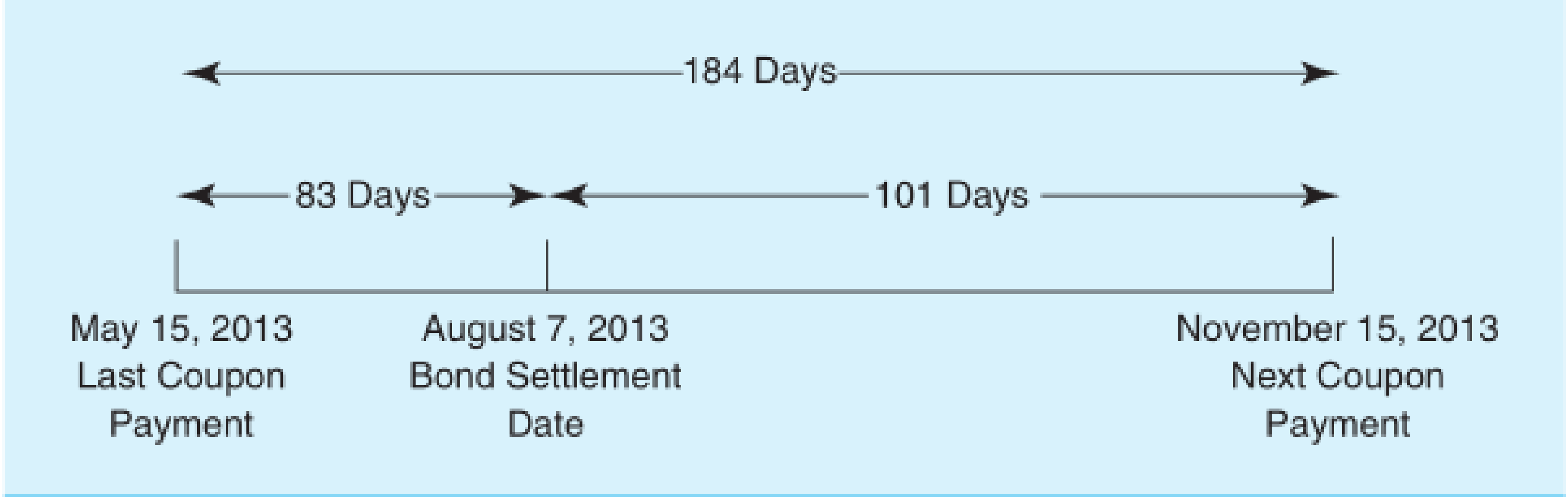
$$= 110.5954$$

or to the nearest $\frac{1}{32}$, 110 $\frac{19}{32}$. The asked quote reported in *The Wall Street Journal Online* was indeed 110 $\frac{19}{32}$.

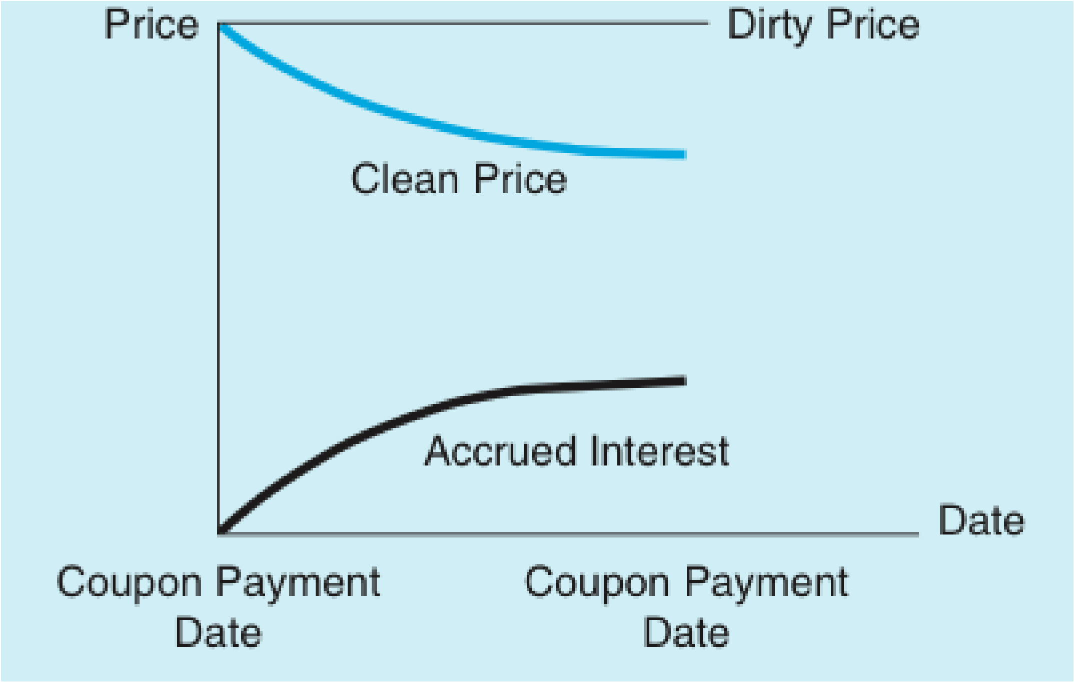
Dirty price/clean price

- At settlement, the buyer must pay the seller the purchase price of the T-note or T-bond plus accrued interest. The sum of these two is often called the *full price* or *dirty price* of the security. The price without the accrued interest added on is called the *clean price.*

Figure 6–5 Time Line Used to Determine Accrued Interest on a Bond



$$\text{Accrued interest} = \frac{INT}{2} \times \frac{\text{Actual number of days since last coupon payment}}{\text{Actual number of days in coupon period}}$$


Price
Dirty Price
Clean Price
Accrued Interest
Date
Coupon Payment Date
Coupon Payment Date

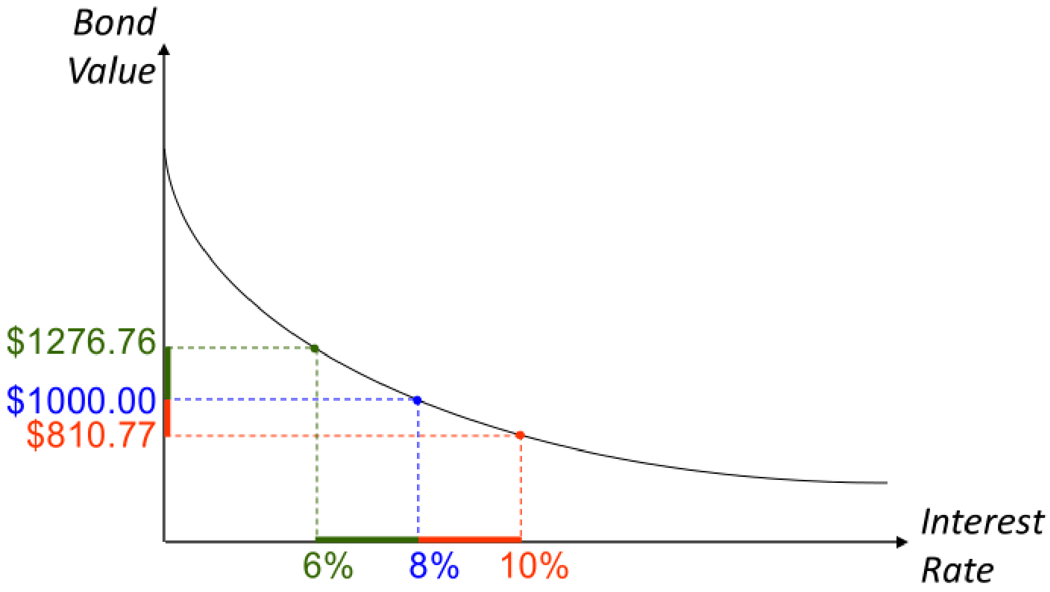
Fundamental Bond relations

- 1. Bond yields vary inversely with changes in bond prices.
- 2. Bond price volatility increases as maturity increases.
- 3. Bond price volatility decreases as coupon rates increase.

e.g. Yield decrease with bond price

Bond Prices and Interest Rates

Bond price is an **inverse** and **convex** function of interest rates



E.g. Volatility increases with maturity

		Price Change if Yield Changes to 6 Percent			Price Change if Yield Changes to 4 Percent		
(1) Maturity (years)	(2) Bond Price at 5 percent Yield ($)	(3) Bond Price ($)	(4) Loss from Increase in Yield ($)	(5) Price Volatility (percent)	(6) Bond Price ($)	(7) Gain from Decrease in Yield ($)	(8) Price Volatility (percent)
1	$1,000	$990.57	$ 9.43	-0.94%	$1,009.62	$ 9.62	0.96%
5	1,000	957.88	42.12	-4.21	1,044.52	44.52	4.45
10	1,000	926.40	73.60	-7.36	1,081.11	81.11	8.11
20	1,000	885.30	114.70	-11.47	1,135.90	135.90	13.59
40	1,000	849.54	150.46	-15.05	1,197.93	197.93	19.79
100	1,000	833.82	166.18	-16.62	1,245.05	245.05	24.50

E.g. Volatility decreases with coupon rate

	10 year maturity	Price Change if Yield Changes to 6 Percent			Price Change if Yield Changes to 4 Percent		
(1) Coupon Rate (percent)	(2) Bond Price at 5 percent Yield ($)	(3) Bond Price ($)	(4) Loss from Increase in Yield ($)	(5) Price Volatility (percent)	(6) Bond Price ($)	(7) Gain from Decrease in Yield ($)	(8) Price Volatility (percent)
0%	$613.91	$ 558.39	$ 55.52	-9.04%	$ 675.56	$ 61.65	10.04%
5	1,000.00	926.40	73.60	-7.36	1,081.11	81.11	8.11
10	1,386.09	1,294.40	91.69	-6.62	1,486.65	100.56	7.25

Macaulay's Duration

- Duration (or Macaulay's Duration) is the weighted average of the time to receipt of the cash flows with the weights proportional to the present value of the payment
  - A measure of the effective maturity of a bond.
  - Formula for Macaulay's Duration:
  - where the wei

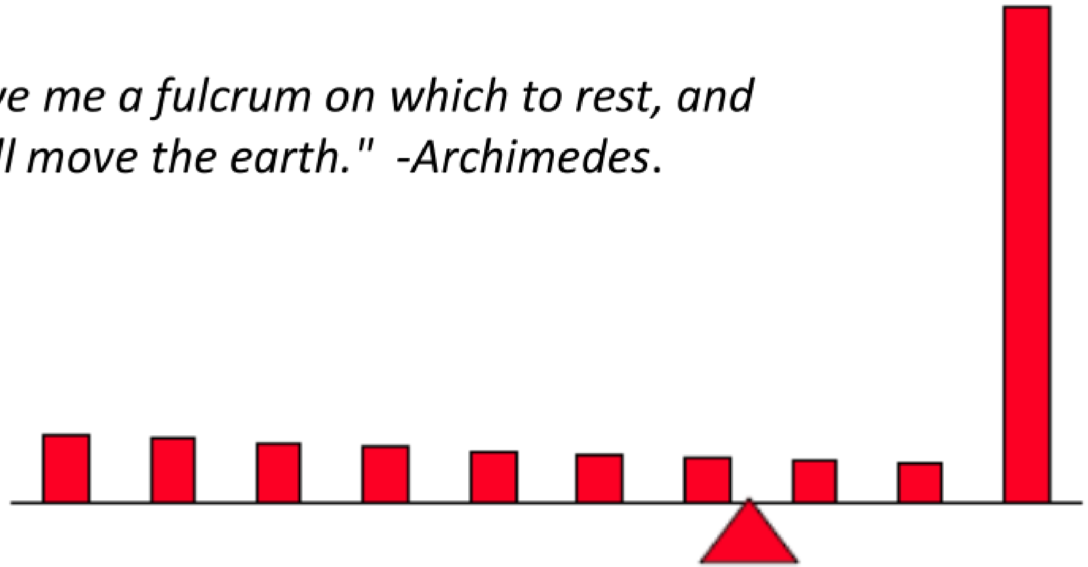
$$D = \sum_{t=1}^{T} w_t t = w_1 \times 1 + w_2 \times 2 + \cdots + w_T \times T$$

$$w_t = \frac{PV(CF_t)}{Bond\ Price} = \frac{CF_t \big/ (1+y)^t}{Bond\ Price}$$

Fulcrum Analogy

- Think of the following bars as weights on a board
- If you try to balance the board, where do you put the fulcrum?

"Give me a fulcrum on which to rest, and I will move the earth." *-Archimedes.*



Properties of Duration

- The greater the duration, the greater is price volatility of the bond for a given change in interest rates.

- All else equal:
  - Bonds with higher coupon rates have shorter durations, all else equal.
  - Bonds with longer maturities have longer durations.
  - The higher the yield to maturity, the shorter is duration.

Duration: An Example

- Find Macaulay's duration (assume a semi-annual YTM of 5% for both bonds):
  - 8% coupon, semi-annual payment bond with 2-year maturity, face value of $1,000
  - Zero-coupon bond, face value of $1,000

A	B	C	D	E	F	G
Semi-annual YTM		Time until		PV of CF		Column (C)
5.00%		Payment		(Discount rate =		times
	Period	(Years)	Cash Flow	5% per period)	Weight	Column (F)
		t	CF_t	$PV(CF_t)$	w_t	$w_t \times t$
A. 8% coupon bond	1	0.5	40	38.095	0.0395	0.0197
	2	1.0	40	36.281	0.0376	0.0376
	3	1.5	40	34.554	0.0358	0.0537
	4	2.0	1040	855.611	0.8871	1.7741
Sum:			Σ	964.540	1.0000	**1.8852**
B. zero coupon	1	0.5	0	0.000	0.0000	0.0000
	2	1.0	0	0.000	0.0000	0.0000
	3	1.5	0	0.000	0.0000	0.0000
	4	2.0	1000	822.702	1.0000	2.0000
Sum:			Σ	822.702	1.0000	**2.0000**

Aside: Deriving Macaulay Duration

- Bond price is the sum of discounted cash flows

$$P = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}$$

- Take a partial derivative w.r.t. $(1 + y)$

$$\frac{\partial P}{\partial(1+y)} = -\sum_{t=1}^{T} t \times \frac{CF_t}{(1+y)^{t+1}} = -\frac{1}{(1+y)} \sum_{t=1}^{T} t \times \frac{CF_t}{(1+y)^t}$$

$$\frac{\Delta P}{\Delta(1+y)} \approx -\frac{1}{(1+y)} \sum_{t=1}^{T} t \times \frac{CF_t}{(1+y)^t}$$

- Divide by P and rearrange

$$\frac{\Delta P}{P} \approx -\frac{\Delta(1+y)}{(1+y)} \sum_{t=1}^{T} t \times \frac{CF_t/(1+y)^t}{P}$$

$$= -\frac{\Delta(1+y)}{(1+y)} \sum_{t=1}^{T} t \times w_t \Rightarrow \frac{\Delta P}{P} \approx -D\frac{\Delta(1+y)}{(1+y)}$$

- Interpretation: the proportional price change approximately equals the proportional change in $(1 + y)$ times the bond's Macaulay duration

An Example

- Consider an 8% coupon, annual payment bond with 3-year maturity, face value of $1000, YTM=10%
  - It is selling at $950.263 and its Macaulay duration is 2.777
- Suppose yield rises by 50 basis points (0.5%). How much does the bond price change?
  - Exact:
    New price = $938.372 (use calculator)
    Price change = $938.372−$950.263 = −$11.891
    So the exact % change is −$11.891/$950.263 = −1.25%
  - Approximate price change using duration:
    price change = −2.525x0.5%x$950.263 = −$11.996
    New price using duration approximation is $950.263−$11.996 = $938.267
    The approximate % change is −$11.996/$950.263 = −1.26%

Risk Management: Immunization

- Immunization: a strategy that matches the durations of assets and liabilities to shield net worth from interest rate movements
  - An insurance company issues a GIC (guaranteed investment contract) for $10,000 maturing in 5 years (like a **zero** coupon bond) with a guaranteed interest rate of 8%
  - Obligation: The insurance company must pay $10,000 x(1.08)^5 = $14,693.28 at maturity
  - Suppose the company chooses to fund its GIC obligation by investing $10,000 in 8% **annual** coupon bonds, selling at par value, with 6 years to maturity
    - That is, the company buys the 6-year coupon bond now, and sell it in 5 years
  - Can the sale proceeds fulfill the GIC obligation?

- Check what will happen when yield goes up/down by 1%.

Base Scenario Projections

(6-year coupon bond, coupons reinvested, bond sold at the end of 5 years)

Payment	Years Left	Formula	Future Value	of All Bond Cash Flows
Rates stay at 8%				
1	4	$800 \times (1.08)^4 =$	1088.39	1st Coupon Re-invested 4 years.
2	3	$800 \times (1.08)^3 =$	1007.77	2nd Coupon Re-invested 3 years.
3	2	$800 \times (1.08)^2 =$	933.12	3rd Coupon Re-invested 2 years.
4	1	$800 \times (1.08)^1 =$	864.00	4th Coupon Re-invested 1 year.
5	0	$800 \times (1.08)^0 =$	800.00	5th Coupon Received in 5 years.
Sale	0	10800/1.08 =	10000.00	Bond Sale Proceeds in 5 years.
			14693.28	Future value of All Payments

Compared to **14693.28** (the GIC value at maturity: the GIC is exactly funded with the 6-year bond)

- Under the base scenario, the 6-year 8% coupon bond forms a duration-matching strategy and the GIC obligation is fully funded in 5 years

Yield Falls to 7%

Compared to 10000 (price risk **gain**)

Compared to 1088.39 (re-investment risk **loss**)

Payment	Years Left				
Rates fall to 7%					
1	4	$800 \times (1.07)^4$	=	1048.64	1st Coupon Re-invested 4 years.
2	3	$800 \times (1.07)^3$	=	980.03	2nd Coupon Re-invested 3 years.
3	2	$800 \times (1.07)^2$	=	915.92	3rd Coupon Re-invested 2 years.
4	1	$800 \times (1.07)^1$	=	856.00	4th Coupon Re-invested 1 year.
5	0	$800 \times (1.07)^0$	=	800.00	5th Coupon Received in 5 years.
Sale	0	10800/1.07	=	10093.46	Bond Sale Proceeds in 5 years.
				14694.05	Future value of All Payments

Compared to **14693.28** **(approximately equal)**

- The capital gain at the end of year 5 makes up for the lower re-investment rate of the coupons
- The GIC is still fully funded in 5 years

Yield Increases to 9%

Payment	Years Left				
Rates increase to 9%					
1	4	$800 \times (1.09)^4$	=	1129.27	1st Coupon Re-invested 4 years.
2	3	$800 \times (1.09)^3$	=	1036.02	2nd Coupon Re-invested 3 years.
3	2	$800 \times (1.09)^2$	=	950.48	3rd Coupon Re-invested 2 years.
4	1	$800 \times (1.09)^1$	=	872.00	4th Coupon Re-invested 1 year.
5	0	$800 \times (1.09)^0$	=	800.00	5th Coupon Received in 5 years.
Sale	0	10800/1.09	=	9908.26	Bond Sale Proceeds in 5 years.
				14696.03	Future value of All Payments

Compared to 10000 (price risk **loss**)

Compared to 1088.39 (re-investment risk **gain**)

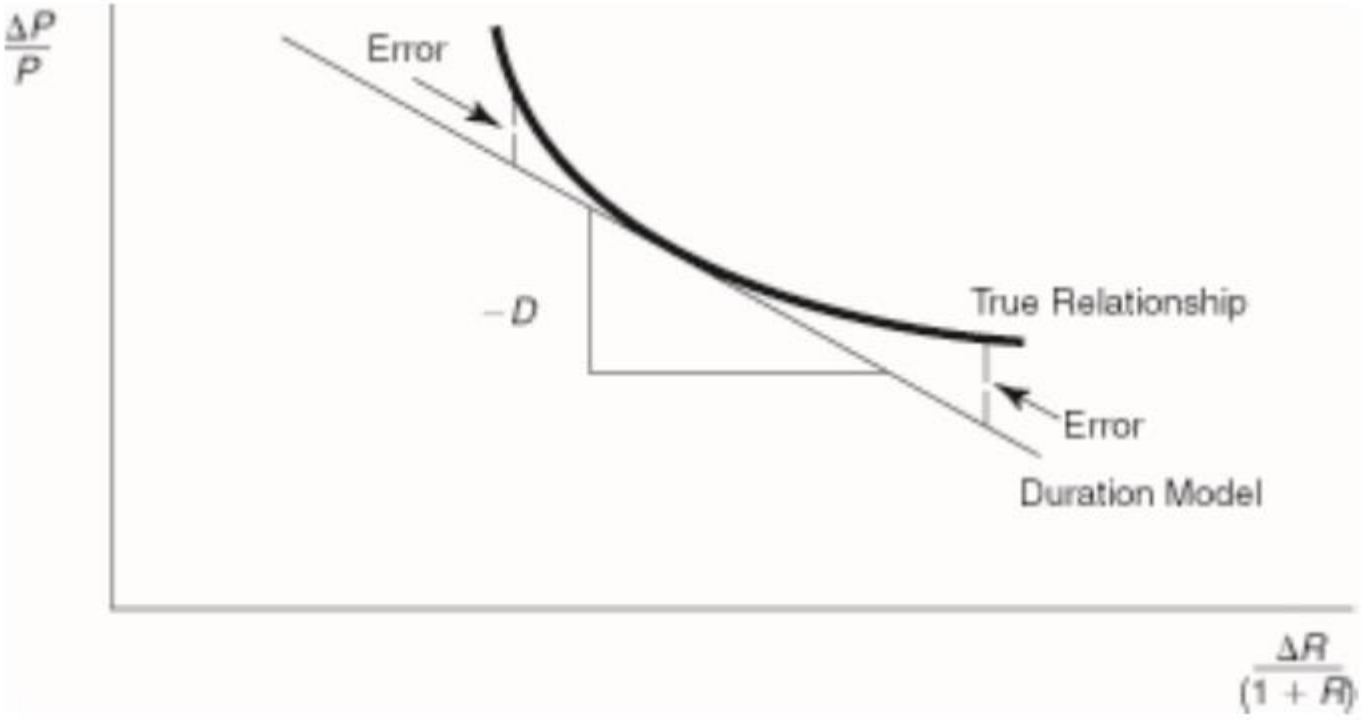
Compared to **14693.28** **(approximately equal)**

- The capital loss at the end of year 5 is covered by the higher re-investment rate of the coupons
- The GIC is still fully funded in 5 years

Building an Immunized Portfolio
Steps Toward Immunization

- Step 1: Find the duration of the liability
  - A one-payment obligation with 7-year duration
- Step 2: Find the duration of the asset portfolio
  - The duration of the 3-year zero-coupon bond is 3 years
- – The duration of the 11-year zero-coupon bond is 11 years
- Asset duration = w × 3 years + (1 - w) × 11 years
  - Note: Duration is additive (very useful!)

Bond price curve



Convexity

- The duration approach for estimating the % change in bond price works well for small changes in interest rates, but not for large changes in interest rates.
- The formula can be modified to work well for large interest changes and the modification is an adjustment for convexity.
- Convexity incorporates the curvature of the price-yield curve into the estimated percentage price change of a bond given an interest rate change:

Convexity effect on price changes

The formula for convexity is:

$$\text{Convexity} = \frac{\sum_{t=1}^{n} \frac{t \times (t+1) \times CF_t}{(1+r)^{(t+2)}}}{\text{P}}$$

The formula for using duration and convexity to estimate the percent change in a bond's price is:

$$\frac{\Delta P}{P} \cong -D\left[\frac{\Delta r}{(1+r)}\right] + \frac{1}{2}(\text{convexity})(\Delta r^2)$$

Exercises

Exercise

- 1. A bond has a market price that exceeds its face value. Which of the following features currently apply to this bond?
  I. discounted price
  II. premium price
  III. yield-to-maturity that exceeds the coupon rate
  IV. yield-to-maturity that is less than the coupon rate
  A. III only
  B. I and III only
  C. I and IV only
  D. II and III only
  E. II and IV only

- **<u>E.</u>** II and IV only

Bond Selling at . . .	Satisfies This Condition
Discount	Coupon Rate < Current Yield < YTM
Premium	Coupon Rate > Current Yield > YTM
Par Value	Coupon Rate = Current Yield = YTM

- 2. Which one of the following relationships is stated correctly?
  A. The coupon rate exceeds the current yield when a bond sells at a discount.
  B. The call price must equal the par value.
  C. An increase in market rates increases the market price of a bond.
  D. Decreasing the time to maturity increases the price of a discount bond, all else constant.
  E. Increasing the coupon rate decreases the current yield, all else constant.

- **D.** Decreasing the time to maturity increases the price of a discount bond, all else constant.

- 3. The bonds issued by Stainless Tubs bear a 6 percent coupon, payable semiannually. The bonds mature in 11 years and have a $1,000 face value. Currently, the bonds sell for $989. What is the yield to maturity?
  A. 5.87 percent
  B. 5.92 percent
  C. 6.08 percent
  D. 6.14 percent
  E. 6.20 percent

$$\$989 = \frac{0.06 \times \$1{,}000}{2} \times \left\{ \frac{1 - \left[1 / \left(1 + \frac{r}{2} \right)^{11 \times 2} \right]}{\frac{r}{2}} \right\} + \frac{\$1{,}000}{\left(1 + \frac{r}{2} \right)^{11 \times 2}}$$

This cannot be solved directly, so it's easiest to just use the calculator method to get an answer. You can then use the calculator answer as the rate in the formula just to verify that your answer is correct.

Enter	11×2	/2	-989	60/2	1,000
	N	I/Y	PV	PMT	FV
Solve for		6.14			

- 4. The Corner Grocer has a 7-year, 6 percent annual coupon bond outstanding with a $1,000 par value. The bond has a yield to maturity of 5.5 percent. Which one of the following statements is correct if the market yield suddenly increases to 6.5 percent?
  A. The bond price will increase by $57.14.
  B. The bond price will increase by 5.29 percent.
  C. The bond price will decrease by $53.62.
  D. The bond price will decrease by 5.43 percent.
  E. The bond price will decrease by 5.36 percent.

$$P = (0.06 \times \$1{,}000) \times \left\{ \frac{1 - \left[1/(1 + 0.055)^7 \right]}{0.055} \right\} + \frac{\$1{,}000}{(1 + 0.055)^7} = \$1{,}028.41$$

Enter	7	5.5		60	1,000
	N	I/Y	PV	PMT	FV
Solve for			-1,028.41		

$$P = (0.06 \times \$1{,}000) \times \left\{ \frac{1 - \left[1/(1 + 0.065)^7 \right]}{0.065} \right\} + \frac{\$1{,}000}{(1 + 0.065)^7} = \$972.58$$

Enter	7	6.5		60	1,000
	N	I/Y	PV	PMT	FV
Solve for			-972.58		

Difference in prices = \$972.58 - \$1,028.41 = -\$55.83

Percentage difference in prices = $\frac{\$972.58 - \$1{,}028.41}{\$1{,}028.41} = -5.43$ percent

Next Class

- Reading Chapter 5 for Money Market
- Reading Chapter 9 for FX Market