Subjective or Objective? Nonparametric Estimation Of Misreporting and Mis-assessment in Corporate Credit Rating*

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19 April 2013

*Comments and suggestions are welcomed.

Abstract

This paper investigates the misreporting and mis-assessment of corporate credit ratings by credit rating agencies (CRAs). We distinguish between "mis-assessment", which is the noise from the unobservable true rating to the rating perceived by CRAs (the internal rating), and "misreporting", which is the difference between perceived and reported rating by CRAs.Using a sample of corporate credit ratings during 1986-2011, we find that the mis-assessment in credit rating is very small and statistically insignificant. Also, there is a U-shaped relationship between true credit rating and misreporting probability. Specifically, CRAs misreport the credit ratings for high-grade firms with a probability of 3%, for middle-grade firms with a probability of 0, and for low-grade firms with a probability of 6%. Second, the misreporting behavior of CRAs differ significantly across the industries. The financial industry has the highest misreporting probability (35% in the lowest-grade firms) and the largest misreporting magnitude (rating grade jump between true and reported grade). The energy industry has the lowest misreporting probability. Last, when economic conditions are bad, the credit rating agent is more likely to deflate the rating.

Key Words: Credit Rating, Misreporting, Mis-assessment, Nonparametric

1 Introduction

Credit ratings play a significant and increasingly important role in borrowers' access to capital and in federal and state legislation. Despite their central role in the financial market, credit rating agencies (hereafter CRAs) have been confronted with continuing criticism regarding the quality of their ratings. Since CRAs collect fees from issuers that they rate instead of the end users of credit ratings such as investors, CRAs have incentive to inflate the rating to please the issuers. Thus, CRAs face the trade-off between the short-term profits from inflating the ratings and the long-term reputation loss. On one hand, some papers (Klein and Leffler (1981); Shapiro (1983)) argue that reputation concerns should discourage CRAs from engaging in rating inflation. On the other hand, some theoretical papers suggest that CRAs may intentionally inflate or deflate the rating. For example, Faure-Grimaud, Peyrache and Quesada (2009) show that issuers may prefer to suppress their ratings if the information content of the rating is too noisy. They also find that competition between rating agencies can result in less information disclosure. Bolton, Freixas and Shapiro (2012) model the competition among CRAs and suggest that ratings are more likely to be inflated during booms and when investors are more trusting. The conflict of theoretical predictions raises an interesting but empirically challenging question: Do credit rating agencies intentionally misreport (inflate/deflate) ratings and, if yes, what is the pattern?

In this paper, we investigate the likelihood and magnitude of intentional misreporting behavior in corporate ratings. Misreporting is defined as an intentional deviation of the reported rating relative to the internal ratings of the borrower that is free from any conflicts of interest. In the context of this paper, we consider inflation as well as deflation as misreporting in our paper. [1] This is an empirically challenging problem as we do not have access to the internal rating given by the CRA, only the externally reported ratings. Further complicating this is the fact that the internal rating may also differ from the true rating, defined as the ratings that the obligor would obtain in the absence of any inflation or deflation, *as well as in the absence of any assessment error on the part of the CRA*.

Our main contribution is that we propose a latent variable approach to infer the misreporting behavior of CRAs. Specifically, we treat the reported corporate credit rating as a function of the underlying unobserved true credit rating. We then impose a structure on the assessment and reporting process and the dynamics of the underlying latent true credit rating. We then identify two sources of difference between the true rating and the reported rating by CRAs. The first source is the "mis-assessment", which is the noise from the unob-

[1] For example, if the internal rating is AA and reported rating is AAA, the reported rating is inflated; if the reported ratings is A, the reported rating is deflated.

servable true rating to the rating perceived by CRAs (the internal rating). This is because the CRAs may not 100% accurately infer the true quality of the firms. The second source is "misreporting", which is the difference between the rating perceived by CRAs and the rating reported by CRAs.

Using a sample of corporate credit ratings during 1986-2011, we find that there is a U-shaped relationship between true credit rating and misreporting probability. Specifically, CRAs misreport the credit ratings for high-grade firms with a probability of 3%, for middle-grade firms with a probability of 0, and for low-grade firms with a probability of 6%. Understanding the properties of misreporting behavior is important, given that credit ratings play a significant and increasingly important role in borrowers' access to capital and in federal and state legislation. Since we do not observe the true rating, we can not compare it with reported rating by CRAs and directly assess the misreporting probability. Our result implies that, using reported rating directly may understate the downward risk in the low rating grade.

Second, we investigate the heterogeneity of the misreporting behavior across different industries. We find that the financial industry has highest misreporting probability (35% in the lowest rating group) and misreporting magnitude (rating grade jump between true and reported grade) and energy industry has lowest misreporting probability. Altman and Kao (1992) and Bangia, Diebold, Kronimus, Schagen and Schuermann (2002) document that rating transition matrices vary with the industries of obligor. This effect would be due to the different incentive in misreporting the rating across industries. Those industries which have larger number of issuance would be more attractive to the CRAs. In order to get the rating deals, CRAs may have higher incentive to inflate the rating. Although our result do not directly answer this question, we document a significant industry heterogeneity in misreporting behaviors of CRAs.

Third, we investigate the heterogeneity of the misreporting behavior in different stages of business cycle. When the economic condition is good, the credit agent is more likely and heavily to inflate the rating. Nickell, Perraudin and Varotto (2000) identify the incremental impact of industry, domicile, and stage of the business cycle on the distribution of ratings changes. Our paper suggest that, at least partially, the misreporting behavior may count for the observed changing in distribution of rating in different stages of business cycle.

Last, the CRAs in general have quite small mis-assessments. In most of cases, the mis-assessments is not statistically difference from 0 for all the rating grades except the rating worse than "B" rating. This is not surprising given that CRAs have been in corporate rating area for decades and they have ample of experience in this industry.

Our study contributes to the credit rating accuracy literature. To our knowledge, this is the first study to explicitly examine the likelihood and magnitude of the misreporting in corporate credit rating and separate it out from "mis-assessment". Prior literature tends to use some specific events to evaluate their impact on rating quality. For example, Becker and Milbourn (2011) documents a negative association between rating quality and the entry of a third rating agency-Fitch. This finding suggests that competition in the ratings industry may reduce the incumbents' future economics rents, and thereby, weaken reputation incentives for quality provision. Xia (2012) finds a significant improvement in the quality of S&P's ratings following rating initiation of Egan-Jone Rating Company which is an investor-paid rating agency. One exception is Griffin and Tang (2012) which use the output of a leading CRA's main quantitative model. They find that a top credit rating agency frequently made positive adjustment beyond its main model for collateralized debt obligations (CDOs). In this paper, we impose the structure of the rating change and we are able to use the whole rating sample and then directly estimate the misreporting probability.

Our work is also related to literature that examines the transition of credit ratings. Altman and Kao (1992) and Bangia et al. (2002) document that rating transition matrices vary with the stages of the business cycle, the industries of obligor and the length of time that has elapsed since the bond issuance. Frydman and Schuermann (2008) propose a parsimonious model that is a mixture of two Markov chains. While all these papers are based on the reported ratings to calculate the transition probability of the credit rating, our paper explicitly models the misreporting behavior and estimate the underlying transition probability matrix. We also allow the misreporting behavior and the underlying transition probability matrix of credit ratings to be different across industry and business cycle.

The rest of the paper is organized as follows. Section 2 provides theoretical results on the identification and estimation of the misreporting probabilities, mis-assessment of CRAs, and the marginal distribution of the underlying credit rating. Section A reports the result for testing the consistency of the estimators proposed using simulation. Section 3 illustrates the data structure. Section 4 presents our main empirical results on the estimated misreporting probabilities and the corrected rating category distribution. The last section concludes. [2].

2 A closed-form identification and estimation

In this section, we presents a closed-form identification and estimation procedure for the misreporting and mis-assessment distributions. The key idea is to use reported ratings across periods to identify the total error and the transition matrices. Then we use rating outlooks as an IV to identify the misreporting matrix. Note that the total error is a mixture

[2] More robustness check on the validity of this method is available upon request

of mis-assessment and misreporting. The identification of the mis-assessment matrix then follows as the latter two are identified.

2.1 Model setup

Let R_t^* denote the latent true rating at period t. The CRAs may not 100% accurately infer the true quality of the firms. Therefore, the CRA perceive a internal rating r_t^*, which is the true rating R_t^* plus some error which we denote as "mis-assessment". After CRAs observe the internal rating r_t^*, they decide to report the rating r_t. The difference between the perceived rating and reported rating by CRAs is denoted as the "misreporting". Figure 1 demonstrates the model setup.

[Figure 1 about here]

2.2 Assumptions and identification

This section presents the identification strategy and provides detail economic intuition behind each assumption. Mis-assessment and misreporting can be separately and uniquely identified if the assumptions provided are satisfied.

Assumption 1 *Conditional on firms' characteristics X and current true rating R_t^*, the ratings that are more than one year ago, beyond the current credit rating, has no predictive power over the true rating in the next year: $Pr(R_{t+12}^*|R_t^*, R_{t-12}^*, X) = Pr(R_{t+12}^*|R_t^*, X)$, for all t.*

The intuition is that credit ratings convey the entity's future debt paying quality based on current information. It does not depend on the historical ratings. This is easy to verify based on the definition of the credit ratings. The definition of credit ratings in S&P website[3] says that *"Credit ratings are forward-looking opinions about credit risk. Standard & Poor's credit ratings express the agency's opinion about the ability and willingness of an issuer, such as a corporation or state or city government, to meet its financial obligations in full and on time."* The reason we choose 12 months lag is based on the fact that CRAs normally reevaluate the rating once a year unless significant corporate events occur, such as merger and acquisition. We use data one year apart to mitigate the influence of past reported rating on future reported rating when conditional on observed rating. This is to alleviate the concern of time series correlation of the reported ratings. Therefore, our model allow the autocorrelation between credit rating to be up to 11 time steps.

[3] http://www.standardandpoors.com/ratings/definitions-and-faqs/en/us

Assumption 2 *Conditional on the firms' characteristics X, the distribution of the rating that the agency observes only depends on current true underlying status:* $Pr(r_t^*|R_t^*, X, r_{\neq t}^*, R_{\neq t}^*) = Pr(r_t^*|R_t^*, X)$ *for all t.*

This assumption implies past or future true ratings does not play a role in the mis-assessment distribution during the information obtaining process, given the current true rating. The latent true rating and firms' characteristics together determine the mis-assessment distribution. In other words, mis-assessment distributions are independent when conditional on the true rating, but they are allowed to be different for each rating category.

Assumption 3 *Conditional on the firms' characteristics X, the misreporting distribution only depends on the status that the agency observes:* $Pr(r_t|R_t^*, r_t^*, X, R_{\neq t}^*, r_{\neq t}^*) = Pr(r_t|r_t^*, X)$ *for all t.*

This assumption implies that past or future perceiving ratings and reported ratings do not matter when CRAs report the rating conditional on current observing rating. The perceiving rating and firms' characteristics together determine the reported rating. In other words, the reporting behavior in each period is uncorrelated. We allow the misreporting behavior to depend on the perceiving rating status, saying the misreporting distribution could be different for each rating category.

Under Assumption 1, 2 and 3, we can associate the observed joint rating distribution r_{t+12}, r_t, r_{t-12} with the total error (mis-assessment plus misreporting) distribution through following:

by law of total probability:

$$
\begin{aligned}
& pr(r_{t+12}, r_t, r_{t-12}|X) \\
= & \sum pr(r_{t+12}, r_{t+12}^*, R_{t+12}^*, r_t, r_t^*, R_t^*, r_{t-12}, r_{t-12}^*, R_{t-12}^*|X) \\
= & \sum pr(r_{t+12}|r_{t+12}^*, X) pr(r_{t+12}^*|R_{t+12}^*, X) pr(R_{t+12}^*|R_t^*, X) pr(r_t|r_t^*, X) \\
& pr(r_t^*|R_t^*, X) pr(r_{t-12}|r_{t-12}^*, X) pr(r_{t-12}^*|R_{t-12}^*, X) pr(r_t^*, R_{t-12}^*|X) \\
= & \sum pr(r_t|R_t^*, X) pr(r_{t+12}|R_t^*, X) pr(R_t^*, r_{t-12}, X)
\end{aligned}
$$

sum over r_{t+12}, we then have

$$
pr(r_t, r_{t-12}|X) = \sum pr(r_t|R_t^*, X) pr(R_t^*, r_{t-12}|X)
$$

Following the identification results in Hu (2008) we show that identification of the total error distribution ($Pr(r_t|R_t^*, X)$) is feasible. In order to apply this novel method, we introduce matrix notation.

For any given subgroup with firm characteristics X=x, and fix the future rating to be some certain grade, i.e. $R_{t+12}^* = k \in \{1, 2, ..K\}$, matrix are defined as follows:

$$P_{r_t|R_t^*,x} \equiv \begin{pmatrix} Pr(r_t = 1|R_t^* = 1, X = x) & Pr(r_t = 1|R_t^* = 2, X = x) & ... & Pr(r_t = 1|R_t^* = K, X = x) \\ Pr(r_t = 2|R_t^* = 1, X = x) & Pr(r_t = 2|R_t^* = 2, X = x) & ... & Pr(r_t = 2|R_t^* = K, X = x) \\ . & . & . & . \\ Pr(r_t = K|R_t^* = 1, X = x) & Pr(r_t = K|R_t^* = 2, X = x) & ... & Pr(r_t = K|R_t^* = K, X = x) \end{pmatrix}$$

$$P_{r_t,r_{t-12}|x} \equiv \begin{pmatrix} Pr(r_t = 1, r_{t-12}^* = 1|X = x) & ... & ... & Pr(r_t = 1, r_{t-12}^* = K|X = x) \\ Pr(r_t = 2, r_{t-12}^* = 1|X = x) & ... & ... & Pr(r_t = 2, r_{t-12}^* = K|X = x) \\ . & . & . & . \\ Pr(r_t = K, r_{t-12}^* = 1|X = x) & ... & ... & Pr(r_t = K, r_{t-12}^* = K|X = x) \end{pmatrix}$$

$$P_{R_t^*,r_{t-12}|x} \equiv \begin{pmatrix} Pr(R_t^* = 1, r_{t-12} = 1|X = x) & ... & ... & Pr(R_t^* = 1, r_{t-12} = K|X = x) \\ Pr(R_t^* = 2, r_{t-12} = 1|X = x) & ... & ... & Pr(R_t^* = 2, r_{t-12} = K|X = x) \\ . & . & . & . \\ Pr(R_t^* = K, r_{t-12} = 1|X = x) & ... & ... & Pr(R_t^* = K, r_{t-12} = K|X = x) \end{pmatrix}$$

$$P_{k,r_t,r_{t-12}|x} \equiv \begin{pmatrix} Pr(r_{t+12} = k, r_t = 1, r_{t-12} = 1|X = x) & ... & Pr(r_{t+12} = k, r_t = 1, r_{t-12} = K|X = x) \\ Pr(r_{t+12} = k, r_t = 2, r_{t-12} = 1|X = x) & ... & Pr(r_{t+12} = k, r_t = 2, r_{t-12} = K|X = x) \\ . & . & . \\ Pr(r_{t+12} = k, r_t = K, r_{t-12} = 1|X = x) & ... & Pr(r_{t+12} = k, r_t = K, r_{t-12} = K|X = x) \end{pmatrix}$$

$$D_{k|R_t^*,x} \equiv \begin{pmatrix} Pr(r_{t+12} = k|R_t^* = 1, X = x) & 0 & ... & 0 \\ 0 & Pr(r_{t+12} = k|R_t^* = 2, X = x) & ... & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & ... & Pr(r_{t+12} = k|R_t^* = K, X = x) \end{pmatrix}$$

Consequently, we can obtain the following two equation in matrix form:

$$P_{k,r_t,r_{t-12}|x} = P_{r_t|R_t^*,x} D_{k|R^*,x} P_{R_t^*,r_{t-12}|x} \tag{1}$$

and

$$P_{r_t,r_{t-12}|x} = P_{r_t|R_t^*,x} P_{R_t^*,r_{t-12}|x} \tag{2}$$

In order to identify the total error through equations 1 and 2, we need to impose following assumptions:

Assumption 4 *for any $X = x$, the matrix $P_{r_t,r_{t-12}|x}$ is invertible.*

This assumption is imposed on the observed density $P_{r_t,r_{t-12}|x}$, and therefore, is directly testable from the data. Given this condition, the matrices $P_{r_t|R_t^*,x}$, and $P_{R_t^*,r_{t-12}|x}$ are both invertible. Consequently, post multiplying $P^{-1}_{r_t,r_{t-12}|x}$ to equation 1 leads to the following key identification equation:

$$P_{k,r_t,r_{t-12}|x}P^{-1}_{r_t,r_{t-12}|x} = P_{r_t|R_t^*,x}D_{k|R_t^*,x}P^{-1}_{r_t|R_t^*,x} \tag{3}$$

The matrix on the left-hand-side can be formed from the observed data. Since $D_{k|R^*,x}$ is a diagonal matrix, the matrix on the right-hand size represents an eigenvalue-eigenvector decomposition of the left-hand-side matrix, with $D_{k|R^*,x}$ as the diagonal matrix consisted of eigenvalues, and $P_{r_t|R_t^*,x}$ as the matrix of eigenvectors correspondingly. The normalization on $P_{r_t|R_t^*,x}$ is given by the fact that every column in $P_{r_t|R_t^*,x}$ should be added up to one because of probability theory. Uniqueness of this decomposition requires the eigenvalues to be distinctive, hence we assume

Assumption 5 *For any $X = x$ and every future reported rating $r_{t+12} = k$, rating agency reports different latent true rating k with different probabilities. i.e., $Pr(r_{t+12} = k|R_t^* = i, X = x)$ are different for different $i \in \{1, 2, ..., K\}$*

Assumption 5 is also testable from equation 3 because they are the eigenvalues of the observed matrix $P_{k,r_t,r_{t-12}|x}P^{-1}_{r_t,r_{t-12}|x}$.

The distinct eigenvalues guarantee the uniqueness of the eigenvectors. Since we do not observe the R_t^* in the sample, we need to reveal the value R_t^* for each eigenvector $Pr(r_t|R_t^* = r_t^*, X = x)$. In other words, the ordering of the eigenvalues or the eigenvectors is still arbitrary in equation 3. In order to eliminate this ambiguity, we make the following assumption.

Assumption 6 *For any x, the rating agency is more likely to report the true rating grater than to report any other possible ratings, i.e.*
$Pr(r_t = i|R_t^* = i, x) > Pr(r_t = j|R_t^* = i, x)$ for $j \neq i$

Although this assumption does not reveal the value of these reporting probabilities, nor require the probability of reporting the truth to be larger than 50%, given that the credit rating is widely used in trading and regulation requirement, we believe that the truly reporting probabilities is larger than 50 % and therefore it is nature to make this assumption. This assumption provides an ordering of the eigenvectors and the eigenvalues in equation 3. Therefore, it leads to a unique decomposition. Thus, the total error distribution matrix $P_{r_t|R_t^*,x}$ is uniquely determined from the observed matrices $P_{k,r_t,r_{t-12}|x}$ and $P_{r_t,r_{t-12}|x}$.

Above first step identification provides us the total error distribution matrix, which is the combination of the mis-assessment distribution and reported behavior.

$$\begin{aligned} pr(r_t|R_t^*, X) &= \sum pr(r_t, r_t^*|R_t^*, X) \\ &= \sum pr(r_t|r_t^*, X) pr(R_t|R_t^*, X) \end{aligned}$$

for every type of firms($X = x$), we can represent the correlation between total error distribution, mis-assessment distribution that affecting what grade the agent observes, and the misreporting behavior of the agent into following matrix:

$$P_{r_t|R_t^*,x} = P_{r_t|r_t^*,x} P_{r_t^*|R_t^*,x}$$

where

$$P_{r_t|r_t^*,x} \equiv \begin{pmatrix} Pr(r_t = 1|r_t^* = 1, X = x) & ... & ... & Pr(r_t = 1|r_t^* = K, X = x) \\ Pr(r_t = 2|r_t^* = 1, X = x) & ... & ... & Pr(r_t = 2|r_t^* = K, X = x) \\ . & . & . & . \\ Pr(r_t = K|r_t^* = 1, X = x) & ... & ... & Pr(r_t = K|r_t^* = K, X = x) \end{pmatrix}$$

$$P_{r_t^*|R_t^*,x} \equiv \begin{pmatrix} Pr(r_t^* = 1|R_t^* = 1, X = x) & ... & ... & Pr(r_t^* = 1|R_t^* = K, X = x) \\ Pr(r_t^* = 2|R_t^* = 1, X = x) & ... & ... & Pr(r_t^* = 2|R_t^* = K, X = x) \\ . & . & . & . \\ Pr(r_t^* = K|R_t^* = 1, X = x) & ... & ... & Pr(r_t^* = K|R_t^* = K, X = x) \end{pmatrix}$$

which indicates separating the mis-assessment distribution and reported behavior requires more information. Following illustrates how to identify misreported probability using additional information. After rating agency perceive the rating status R_t, which is the true status with noise, the agency not only reports the rating, but also provides extra information, rating outlook denoted as w_t, we can separately identify the misreport behavior as follows. The rating outlook is classified as positive, stable, and negative, which we assign value w_t=+1,0,-1, respectively. We state the assumption below:

Assumption 7 *Conditional on firms characteristics (X) and what the agency perceives (r_t^*), the reporting rating r_t and rating outlook w_t are independent, i.e.* $Pr(r_t, w_t|r_t^*, X) = Pr(r_t|r_t^*, X)Pr(w_t|r_t^*, X)$

This assumption basically means conditional on the perceived rating grade, the probability that agents report the grade to be r_t should be the same whether it reports the rating outlook to be positive, negative or zero. This conditional independence of rating and rating outlook help for backing out the misreported behavior of the agency. Here we do not require the agent report the true rating outlook status.

By law of total probability, again we can use the joint distribution of $w_t, r_t, r_{t-12}|X$ to identify the agent's misreported behavior

$$\begin{aligned} pr(w_t, r_t, r_{t-12}|X) &= \sum pr(w_t, r_t, r_{t-12}, R_t|X) \\ &= \sum pr(r_t|r_t^*, X) pr(w_t|r_t^*, X) prob(r_t^*, r_{t-12}|X) \end{aligned}$$

sum over w_t provides us

$$pr(r_t, r_{t-12}|X) = \sum pr(r_t|r_t^*, X) prob(r_t^*, r_{t-12}|X)$$

for firms with characteristics $X = x$ and a typical rating outlook $w_t = w$, matrix representation of these two equations can be written as follows:

$$\begin{aligned} P_{w,r_t,r_{t-12}|x} &= P_{r_t|r_t^*,x} D_{w_t|r_t^*,x} P_{r_t^*,r_{t-12}|x} \\ P_{r_t,r_{t-12}|x} &= P_{r_t|r_t^*,x} P_{r_t^*,r_{t-12}|x} \end{aligned}$$

where

$$P_{w,r_t,r_{t-12}|x} \equiv \begin{pmatrix} Pr(w_t = w, r_t = 1, r_{t-12} = 1|X = x) & ... & Pr(w_t = w, r_t = 1, r_{t-12} = K|X = x) \\ Pr(w_t = w, r_t = 2, r_{t-12} = 1|X = x) & ... & Pr(w_t = w, r_t = 2, r_{t-12} = K|X = x) \\ . & . & . \\ Pr(w_t = w, r_t = K, r_{t-12} = 1|X = x) & ... & Pr(w_t = w, r_t = K, r_{t-12} = K|X = x) \end{pmatrix}$$

$$D_{w|r_t^*,x} \equiv \begin{pmatrix} Pr(w_t = k|r_t^* = 1, X = x) & 0 & ... & 0 \\ 0 & Pr(w_t = k|r_t^* = 2, X = x) & ... & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & ... & Pr(w_t = k|r_t^* = K, X = x) \end{pmatrix}$$

$$P_{r_t^*,r_{t-12}|x} \equiv \begin{pmatrix} Pr(r_t^* = 1, r_{t-12} = 1|X = x) & ... & Pr(r_t^* = 1, r_{t-12} = K|X = x) \\ Pr(r_t^* = 2, r_{t-12} = 1|X = x) & ... & Pr(r_t^* = 2, r_{t-12} = K|X = x) \\ . & . & . \\ Pr(r_t^* = K, r_{t-12} = 1|X = x) & ... & Pr(r_t^* = K, r_{t-12} = K|X = x) \end{pmatrix}$$

Given that we have already imposed the invertibility of matrix $P_{r_t,r_{t-12}}$ in the first step identification, it leads us to the following main identification equation for misreporting behavior:

$$P_{w_t,r_t,r_{t-12}|x} P^{-1}_{r_t,r_{t-12}|x} = P_{r_t|r_t^*,x} D_{w_t|r_t^*,x} P^{-1}_{r_t|r_t^*,x} \tag{4}$$

The matrix on the left-hand-side can be formed from the observed data. Since $D_{w_t|r_t^*,x}$ is a diagonal matrix, the matrix on the right-hand size represents an eigenvalue-eigenvector decomposition of the left-hand-side matrix, with $D_{w_t|r_t^*,x}$ as the diagonal matrix consisted of eigenvalues, and $P_{r_t|r_t^*,x}$ as the matrix of eigenvectors correspondingly. The normalization on $P_{r_t|r_t^*,x}$ is given by the fact that every column in $P_{r_t|r_t^*,x}$ should be added up to one because of probability theory. Uniqueness of this decomposition requires the eigenvalues to be distinctive, hence we assume

Assumption 8 *In each period, conditional on firms characteristics $X = x$ and what the rating agency perceives, the probability that it reports the rating outlook to be positive varies with r_t^*. i.e., $Pr(w_t = +1|r_t^* = i, x)$ varies for different $r_t^* = i \in \{1, 2, ..., K\}$*

Assumption 8 is also testable from equation 4 because they are the eigenvalues of the observed matrix $P_{w_t,r_t,r_{t-12}|x}P^{-1}_{r_t,r_{t-12}|x}$.

Ordering of the eigenvalue requires additional assumption stated as follow:

Assumption 9 *The rating agency is more likely to report the rating it perceives than to report any other possible ratings, i.e.*
$Pr(r_t = i|r_t^* = i, x) > Pr(r_t = j|r_t^* = i, x)$ for $j \neq i$

Although this assumption does not reveal the value of these reporting probabilities, not require the probability of reporting the truth to be larger than 50%, given that the credit rating is widely used in trading and regulation requirement, we believe that the truly reporting probabilities is larger than 50 % and therefore it is nature to make this assumption. This assumption provides an ordering of the eigenvectors and the eigenvalues in equation 3. Therefore, it leads to a unique decomposition. Thus, the total error distribution matrix $P_{r_t|r_t^*}$ is uniquely determined from the observed matrices $P_{w_t=1,r_t,r_{t-12}|x}$ and $P_{r_t,r_{t-12}|x}$.

We summarize the identification results as follows:

Theorem 1 *Under Assumptions 1, 2, 3, 4, 5, 6, 7, 8, 9 the observed joint distribution of the reported rating $Pr(r_{t+12}, r_t, r_{t-12}|x)$ and $Pr(w_t, r_t, r_{t-12}|x)$ uniquely identify the misreporting matrix $Pr(r_t|r_t^*, x)$ and mis-assessment matrix $Pr(r_t^*|R_t^*, x)$, respectively.*

Proof. The theorem is easily proved using Theorem 1 in Hu (2008). ■

Since our identification results are constructive, we can directly follow the identification procedure to form an estimator. Specifically, we may estimate $Pr(r_{t+12}, r_t, r_{t-12}|x)$, $Pr(r_t, r_{t-12}|x)$ and $Pr(w_t, r_t, r_{t-12}|x)$ using sample average for a discrete $x = \overline{x}$. For a continuous x, we may simply use a kernel density estimator [4].

Following the identification procedure, we may estimate the misreporting probability $Pr(r_t|r_t^*, x)$ and the mis-assessment distribution $Pr(r_t^*|R_t^*, x)$. Such an estimator has a closed-form

[4] See appendix for simulation result

expression so that one does not need to use the regular optimization algorithms, which usually need many iterations.

The total error distribution enables us to obtain the distribution of the latent true credit rating status $prob(R^*|x)$ using following equation:

$$Pr(r_t|X) = \sum_{R_t^*} Pr(r_t|R_t^*, X)Pr(R_t^*|X) \tag{5}$$

which equivalently to

$$\begin{pmatrix} Pr(r_t = 1|X) \\ . \\ . \\ . \\ Pr(r_t = K|X) \end{pmatrix} = P_{r_t|R_t^*,X} \begin{pmatrix} Pr(R_t^* = 1|X) \\ . \\ . \\ . \\ Pr(R_t^* = K|X) \end{pmatrix}$$

Consequently, inverting the error distribution matrix and the observed crediting rating leads to:

$$\begin{pmatrix} Pr(R_t^* = 1|X) \\ . \\ . \\ . \\ Pr(R_t^* = K|X) \end{pmatrix} = A^{-1}_{r_t|R_t^*,X} \begin{pmatrix} Pr(r_t = 1|X) \\ . \\ . \\ . \\ Pr(r_t = K|X) \end{pmatrix}$$

Moreover, we can also estimate the probability that when some companies with characteristics X reported by the rating agency as rating r_t but the true rating is actually R_t^* through Bayesian equation:

$$Pr(R_t^*|r_t, X) = \frac{Pr(r_t|R_t^*, X)Pr(R_t^*|X)}{\sum_{R_t^*} Pr(r_t|R_t^*, X)Pr(R_t^*|X)} \tag{6}$$

Additionally, the transition matrix for latent credit rating can be obtained through following matrix transformation.

$$\begin{aligned} prob(r_{t+12}|R_t^*, X) &= \sum_{R_{t+12}^*} prob(r_{t+12}, R_{t+12}^*|R_t^*, X) \\ &= \sum_{R_{t+12}^*} prob(r_{t+12}|R_{t+12}^*, R_t^*, X)prob(R_{t+12}^*|R_t^*, X) \\ &= \sum_{R_{t+12}^*} prob(r_{t+12}|R_{t+12}^*, X)prob(R_{t+12}^*|R_t^*, X) \end{aligned} \tag{7}$$

where $prob(r_{t+12}|R_t^*, X)$ is the eigenvalues from matrix decomposition in the first step. Moreover, we assume stationary environment indicating $prob(r_{t+12}|R_{t+12}^*, X) = prob(r_t|R_t^*, X)$, which is the eigenvector from matrix decomposition in the first step too. Hence, we can obtain the transition distribution again with aid of matrix representation.

$$P_{r_{t+12}|R_t^*,x} = P_{r_{t+12}|R_{t+12}^*,x} P_{R_{t+12}^*|R_t^*,x}$$

where:

$$P_{r_{t+12}|R_t^*,x} \equiv \begin{pmatrix} Pr(r_{t+12}=1|R_t^*=1,X=x) & ... & Pr(r_{t+12}=1|R_t^*=K,X=x) \\ Pr(r_{t+12}=2|R_t^*=1,X=x) & ... & Pr(r_{t+12}=2|R_t^*=K,X=x) \\ . & . & . \\ Pr(r_{t+12}=K|R_t^*=1,X=x) & ... & Pr(r_{t+12}=K|R_t^*=K,X=x) \end{pmatrix}.$$

$$P_{r_{t+12}|R_{t+12}^*,x} \equiv \begin{pmatrix} Pr(r_{t+12}=1|R_{t+12}^*=1,X=x) & ... & Pr(r_{t+12}=1|R_{t+12}^*=K,X=x) \\ Pr(r_{t+12}=2|R_{t+12}^*=1,X=x) & ... & Pr(r_{t+12}=2|R_{t+12}^*=K,X=x) \\ . & . & . \\ Pr(r_{t+12}=K|R_{t+12}^*=1,X=x) & ... & Pr(r_{t+12}=K|R_{t+12}^*=K,X=x) \end{pmatrix}.$$

$$P_{R_{t+12}^*|R_t^*,x} \equiv \begin{pmatrix} Pr(R_{t+12}^*=1|R_t^*=1,X=x) & ... & Pr(R_{t+12}^*=1|R_t^*=K,X=x) \\ Pr(R_{t+12}^*=2|R_t^*=1,X=x) & ... & Pr(R_{t+12}^*=2|R_t^*=K,X=x) \\ . & . & . \\ Pr(R_{t+12}^*=K|R_t^*=1,X=x) & ... & Pr(R_{t+12}^*=K|R_t^*=K,X=x) \end{pmatrix}.$$

The total error distribution matrix is invertible from assumption 4. Consequently, the latent grade transition distribution can be identified through:

$$P_{R_{t+12}^*|R_t^*,x} = P^{-1}_{r_{t+12}|R_{t+12}^*,x} P_{r_{t+12}|R_t^*,x}$$

3 Data and sample description

We construct our rating sample directly from COMPUSTAT North America ratings and RatingsXpress. This database contains detailed information on S&P credit ratings back to 1986[5]. We focus on the long-term rating of the companies from 1986 to 2011, because the long-term ratings measure the forward-looking long-term ability to meet the financial obligation and therefore more suitable for our assumption on the stability of transition matrix of ratings. We use RatingsXpress to obtain the rating outlook information. We use the monthly data of the ratings. Following prior literature, missing credit rating/outlook is replaced using rating/outlook in the previous month.

[5]Although the dataset has data starting from 1973, there is very few observation prior to 1986.

We exclude the ratings with "NM" (Not Meaningful), "Suspend", "SD" (i.e. firms have selectively defaulted on some obligations), "D" (i.e. firms has defaulted on some obligations and S&P believes that it will generally default on most or all obligations), and "NR" (not rated), because these ratings based on some observable facts to the public thus these is no room to misreport it. There are 21 different levels of rating by the rating agency and we group them into 6 categories: All the AAA level rating to be category 1; all the AA level rating to be category 2; All the A level rating to be category 3; category 4 includes all the triple B rating, i.e BBB+, BBB,BBB-; category 5 includes all the double B rating, i.e BB+, BB,BB-; and category 6 consists of all rating left. Panel A of table 1 presents descriptive statistics for firms rated by S&P between 1986 and 2011.

[Table 1 about here]

Firstly, we calculate 1-year unconditional transition matrices using reported ratings. This permit us to relate our results to earlier studies that have performed similar exercises. The basic assumption of this approach is that, for a given sample, the probability of a transition from rating i to j, say, is a constant parameter, p_{ij}. This amounts to saying that, for a given initial rating, transitions to different possible future ratings follow a constant parameter, temporally independent multinomial process. Estimation may then be performed by taking the fraction of occasions in the sample (or sub-sample) on which an obliger starts the year in state i and end it in j. Result is reported in Panel A of Table 2. The table should be read as follow: row variable is the current rating and column variable is the rating 12 months later, so the $cell_{ij}$ represents the transition probability to the rating j 12 month later given the current rating is i. For example, the first row and first column in panel A says that, given the current rating is AAA, the firm stays as AAA with a probability of 90.05%. Our result is quite comparable to that in previous literature (Nickell et al. (2000)).

Secondly, different from previous literature, we estimate the transition probability matrix after correcting for the misreporting and mis-assessment. As Figure 1 demonstrates, after CRAs learn the true rating of the firms, they impose a consistent misreporting probability for each rating groups defined above. Then CRAs report the after-adjustment rating. Using the estimation method in Section 2, Table 2 panel B reports the results. Comparing panel A and panel B, we can find that the reported transition matrix underestimate the probability of retaining the same rating in 12 months. All the diagonal cells in reported matrix (panel A) are smaller than that in panel B. This is not surprise because the transition probability using reported rating neglect the mis-assessment and misreporting. Therefore, these 2 error components contribute to more observed status shifting. Second, the reported transition matrix (panel A) has higher transition probability from higher rating group (AAA, AA, and A) to lower rating groups than that in estimated transition (panel B). Third, the reported transition matrix (panel A) has higher transition probability from lower rating group (BBB, BB&B, and Below B) to higher rating groups than that in estimated transition matrix (panel B). Overall, the reported transition matrix overestimate the probability of credit rating upgrade for low rating groups and underestimate that for high rating groups.

[Table 2 about here]

4 Empirical result

This section estimates the misreporting matrix using observed rating data. We first pool all of the data together and estimate the overall misreportings; we then incorporate firm attributes such as industry, business cycle into the misreporting probability estimation and look into how rating agency behaves differently.

4.1 Misreporting behavior in overall sample

We pool the monthly rating data from 1986 to 2011. In order to build up a consistent sample with that in subsample analysis, we merge the rating data with COMPUSTAT with requirement that the industry information is available [6]. We use the credit watch/rating outlook [7] as additional information besides rating itself to separate out mis-assessment and misreporting. The untabulated result indicates that the mis-assessment is quite small and statistically insignificant in both overall sample and subsample analysis. Therefore, in the following analysis, we take the 0 mis-assessment as the fact and conduct our analysis on misreporting behavior of CRAs.

4.2 Misreporting behavior in overall sample

Table 3 presents the misreporting matrix for the whole sample. The diagonal of matrix stands for the trustfully reporting probability and the cells in upper triangle stand for the inflation probability. The cells in lower triable stand for the deflation probability. We sum up the inflation probability and deflation probability as misreporting probability (not truthfully reporting). We find that, overall, the rating inflation is not that serve in the whole sample, comparing to that in structured financed product as documented in Griffin and Tang (2012). The misreporting probabilities for all rating groups are around 5%. This result could be interpreted as less asymmetric information in corporate rating comparing to that in structured finance products which leads to less room for manipulation of the ratings.

Second, the high rating groups have smaller number of misreporting probability than that for low rating groups. For example, the AAA rated firms would be deflated as AA rating with a probability of 2%, as A rating with a probability of 0.8%, as BBB rating with a probability of 0.2%, as BB&B rating with a probability of 0.1%, and 0 probability for below B rating. In contrast, the Below B rated firms would be inflated as BB&B rating with a probability of 6.5%, as BBB

[6] The estimated misreporting probabilities do not change significantly if we drop this requirement.

[7] We admit that this two variables may not provide much information besides the rating and therefore lead to a weaker identification. But this is the current best proxy we have. Despite the information content of the proxy, in the identification part, we show that our method works very well under our assumptions with the support from simulation result.

rating with a probability of 0.5%, and 0 probability for other ratings. This result is consistent with the facts that the CRAs are reluctant to give the highest ratings. The last row of Table 3 report the total misreporting probability and associated statistical significance. The results are broadly consistent with those in the individual cells in the matrix.

Third, the middle rating groups are less likely to receive a inflated or deflated rating. The magnitude of misreporting for A, BBB, and BB&B rated groups are small and not statistically significant. The possible explanation would be that, since there is a big jump in the BBB investment grade in bond rate, investors would pay more attention to the firms around investment grade and therefore the CRAs are less likely to do the inflation or deflation.

Overall, the misreporting pattern of CRAs shows a U-shape with left side (the high rating groups) having lower misreporting probability (3%), middle having no misreporting, and right side having high misreporting probability (6%). We also test the statistical significance by comparing the estimated matrix to the identity matrix. As shown in the methodology section, the estimated misreporting probability follow the t-distribution.

4.3 Misreporting behavior across industries

As pointed out in Nickell et al. (2000), the transition probability matrix could differ a lot across the industry. Therefore, we would like to check whether the misreporting behavior also differ a lot across the industry. We match the credit rating data with COMPUSTAT and each industry defined by 2-digit Global Industry Classification Standard (GICS), we pool the matched samples to estimate the misreporting probabilities. There are 10 industries in total but we only report the result for 3 industry—energy, consumer staples, and financial industry (result for other industry are available upon request). The reason is that, we are most interested in the financial industry because it suffer the most during the past crisis. We choose energy industry because it is most mature and predictable with least asymmetric information. We randomly pick up the consumer staples from all the other 8 industries.

Table 4 reports the results for 3 industries. There are significant differences across industry for the misreporting behavior. We do not find statistically significant misreporting in energy industry. Meanwhile, the misreporting probability is quite high in financial industry and consumer staples industry. There are some further difference in misreporting between financial firms and consumer staples firms as well. For example, the financial firms are more likely to retain the same rating for high rating groups (95.7%, 97.7%, and 99.2% for financial firms comparing to 90.7%, 93.8%, and 96.4% for consumer staples firms, for AAA, AA, and A respectively). Also, financial firms are more likely to inflate the rating for low rating groups (9.4% and 37.9% comparing to 0.4% and 34.9% for consumer staples firms, for BB&B and Below B respectively). Last the misreporting magnitude (defined as the rating rank jump from true to reported) is higher for financial firms. The largest misreporting magnitude for financial firms is from Below B rating jumping to AA with probability of 35.5%. In contrast, the largest misreporting magnitude for consumer staples firms is from Below rating jumping to BB&B. This result might suggest additional risk might be take into consideration besides the direct use of reported ratings.

Besides the difference of the misreporting behavior across the industries, there exist some consistent patterns across all the industries. The U-shape of the misreporting pattern still holds. The left side of U-shape (the high rating groups) has lower misreporting probability, middle having lowest misreporting probability, and right side has high misreporting probability.

4.4 Misreporting behavior across business cycle

There are some theoretical papers on how the rating inflation might vary across the business cycle. Bolton et al. (2012) model the competition among CRAs and suggest that ratings are more likely to be inflated during booms and when investors are more trusting. Mathis, McAndrews and Rochet (2009) examine the incentives of a monopoly CRA to inflate ratings in a model of endogenous reputation and find that reputation cycles may exist where a CRA build up its reputation by replying information accurately only to exploit this reputation later by collecting fees for inflated ratings.

We formally test the misreporting behavior across business cycle by estimating the misreporting probability in each stage of business cycle. Using the data from NBER's Business Cycle Dating Committee[8], we divide the business cycle into 3 stages—recession, normal, and boom by sorting the yearly real GDP growth rate into tercile. The top tercile with highest real GDP growth rate is defined as boom and the bottom tercile with lowest real GDP growth rate is defined as recession.

Table 5 reports the misreporting result for firms during economic boom and recession. Overall, the total misreporting probability is quite comparable but the pattern is quite different. During economic boom, CRAs are more likely to inflate the rating. Meanwhile, CRAs are more likely to deflate the rating during economic recessions. For example, for the below B rating groups, the firms are inflated with a probability of 35.2% during economic boom comparing to 27.4% in economic recession. For rating groups AA, A, BBB, and BB&B during economic boom, although the misreporting probability is quite similar to that during economic recession, they are more concentrated in inflation instead of deflation.

Overall, our result suggests that, although there are misreporting during economic boom and recession, inflation are more severe when the economic is booming. Our result are more support of the procyclical credit inflation.

5 Robustness checks

In this section, we perform several robustness check to ensure the validity of our estimation method and some more robustness for different subsample analysis.

[8]http://www.nber.org/cycles/recessions.html#navDiv=4

5.1 Evaluation of the assumptions

In this section, we evaluate the key assumptions which are essential to our identification results. We preform extensive Monte Carlo simulations to examine the robustness of our estimator to deviations from Assumption 1, 3. We also test the validity of Assumption 4 and 5 directly using ratings data. For Assumption 6, we argue that it is likely to hold based on prevailing using of the rating in trading and regulation requirement. Although the evaluation result is not reported in the paper, We summarize the main things we have done.

We start with a baseline data generating process (DGP) which satisfies all the maintained assumptions, and show the consistency of our estimator. After that, we allow the DGP deviate from the baseline case to check the robustness of our estimator when each of the assumption is violated.

Assumption 1 imposes conditional independence on the transition of the underlying true credit ratings. In the Monte Carlo simulation setup, we relax this assumption to allow the transition of the true ratings to depend on that 12 months earlier, i.e., $Pr(R^*_{t+12}|R^*_t, R^*_{t-12}) \neq Pr(R^*_{t+12}|R^*_t)$. Our simulation results show that the estimator is robust to reasonable deviations to Assumption 1.

Assumption 3 imposes conditional independence of the misreporting process. We have considered three different kinds of deviations to this assumption in our Monte Carlo simulations. In the first case, we allow the misreportings to be correlated with the latent true ratings 12 months earlier, i.e., $Pr(R_t|R^*_t, X, R_{\neq t}, R^*_{\neq t}) = Pr(R_t|R^*_t, R^*_{t-12}, X)$. In the second case, misreporting may be correlated with the reported ratings 12 months earlier, i.e., $Pr(R_t|R^*_t, X, R_{\neq t}, R^*_{\neq t}) = Pr(R_t|R^*_t, R_{t-12}, X)$. Lastly, we consider a general relaxation, i.e., $Pr(R_t|R^*_t, X, R_{\neq t}, R^*_{\neq t}) = Pr(R_t|R^*_t, R^*_{t-12}, R_{t-12}, X)$. In all cases, our simulation results show that our estimator is robust to reasonable deviations from Assumption 3.

Assumption 4 requires an observed matrix to be invertible, and therefore, is directly testable from the ratings data. We use bootstrapping to show that the determinant of this matrix is significantly different from zero, which implies that the matrix is invertible.

Under Assumption 1, 3 and 4, 5 requires that the eigenvalues of an observed matrix be distinct. We may also directly test this assumption using the ratings data by estimating the difference between the eigenvalues. Our bootstrapping results show that the absolute differences between the eigenvalues are significantly different from zero, which implies that the eigenvalues are distinctive.

Assumption 6 implies that credit rating agency is more likely to report the true rating than any other possible values. We believe this assumption is intuitively reasonable. Also, we are not aware of any studies in the literature that report anything in violation of this assumption.

All the simulation results on the evaluation of assumptions are available upon request.

5.2 Exclusion of recent financial crisis period

We exclude the sample period later than 2006 to check whether the misreporting behavior is due to that in the recent financial crisis. Table 6 reports the results. We find that financial industry has smaller misreporting magnitude before recent financial crisis. For example, for the whole sample, the largest misreporting magnitude for financial firms is from Below B rating jumping to AA with probability of 35.5%. In contrast, before the recent financial crisis, the largest misreporting magnitude for financial firms is from Below rating jumping to BBB with a probability of 34.9%. This result is consistent with the Cornaggia, Cornaggia and Hund (2011) that the inflation is a bigger problem during the recent financial crisis.

6 Conclusion

We use a novel estimation method to explicitly estimate the misreporting probability for each rating groups of the credit rating agency. we treat the reported corporate credit rating as a function of the underlying unobserved true credit rating. We then impose a structure on the misreporting process and the dynamics of the underlying latent true credit rating and identify two sources of difference between the true rating and the reported rating by CRAs. The first error is the "mis-assessment", which is the noise from the unobservable true rating to the rating perceived by CRAs (the internal rating). This is because the CRAs may not 100% accurately infer the true quality of the firms. The second error is "misreporting", which is the difference between the rating perceived by CRAs and the rating reported by CRAs. We find that, overall, the mis-assessment in the credit rating is very small and statistically insignificant. The misreporting pattern of CRAs shows a U-shape with left side (the high rating groups) having lower misreporting probability (3%), middle having no misreporting, and right side having high misreporting probability (6%). Meanwhile, there is a significant difference across rating groups, industry, and business cycle.

Our results contribute to the debate of the credit rating accuracy that the reported rating do not reflect the true rating of the firms. Failure to correct for the misreporting probability might understate the risk of low rating firms. Future research would be extended to measure how this misreporting probability affect the bond price. Also, prior research document the inflated ratings of structured products and their contribution to the financial crisis. Our result shows that there are misreporting outside the crisis period as well.

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A Simulation for testing the consistency of estimator

This section presents some Monte Carlo evidence to support the validation of the method that using eigenvalue-eigenvector decomposition technique to estimate the misreporting probabilities through observed data. Let R_t^* denote the latent true rating in period t with possible values of $\{1,2,3,4,5,6\}$. For the sake of simplicity, we ignore the observed state variables since all the analysis are identical when we condition on firm characteristics. Assuming the underlying rating grade follows a first order Markov Chain with a transition matrix as:

$$prob(R_{t+12}^*|R_t^*, ...R_1^*) = prob(R_{t+12}^*|R_t^*)$$

Providing this transition matrix, we can random draw a series of underlying ratings: $\{R_{t+12}^*, R_t^*, R_{t-12}^*\}_{i=1,...ns}$.

Regarding the mis-assessment distribution that affect the accuracy of what the agent think about the company, we assume it to be i.i.d across time and only depends on the current true rating. The mis-assessment distribution is captured by $prob(r_t^*|R_t^*)$. So sampling the rating CRA perceives in each period t relies on the latent rating and this mis-assessment distribution. From $\{R_{t+12}^*, R_t^*, R_{t-12}^*\}_{i=1,...ns}$ and $prob(r_t^*|R_t^*)$, we can draw $\{r_{t+12}^*, r_t^*, r_{t-12}^*\}_{i=1,...ns}$.

CRA provides the rating r_t and rating outlook w_t for the firm given it observe r_t^* in period t. We assume the reporting behavior is consistent over time, meaning the reporting distributions for both grade and rating outlook are stationary, i.e $prob(r_t|r_t^*) = prob(r_\tau|r_\tau^*)$, for $t \neq \tau$ and $prob(w_t|r_t^*) = prob(r_\tau|r_\tau^*)$, for $t \neq \tau$. With these two probability distributions and $\{r_{t+12}^*, r_t^*, r_{t-12}^*\}_{i=1,...ns}$, we can simulate the rating and the rating outlook that CRA reports, $\{r_{t+12}, r_t, r_{t-12}, w_{t+12}, w_t, w_{t-12}\}_{i=1,...ns}$

Applying our methodology to the simulated data, first we calculate $Pr(r_{t+12} = k, r_t = i, r_{t-12} = j)$ and $Pr(r_t = i, r_{t-12} = j)$ for $k, i, j \in \{1, 2..., K\}$ and then put them into a matrix representation defined in the text. According to key equation:

$$P_{k,r_t,r_{t-12}} P_{r_t,r_{t-12}}^{-1} = P_{r_t|r_t^*} D_{k|r_t^*} P_{r_t|R_t^*}^{-1}$$

Then, we can identify the misreporting distribution from the normalized eigenvectors of matrix $P_{k,r_t,r_{t-12}} P_{r_t,r_{t-12}}^{-1}$

Specifically, we assume the true rating transition matrix and misreporting matrix are as follow:

$$TRUE_{R_{t+12}^*|R_t^*} \equiv \begin{pmatrix} Pr(R_{t+12}^* = 1|R_t^* = 1) & ... & Pr(R_{t+12}^* = 1|R_t^* = 6) \\ Pr(R_{t+12}^* = 2|R_t^* = 1) & ... & Pr(R_{t+12}^* = 2|R_t^* = 6) \\ . & . & . \\ Pr(R_{t+12}^* = 6|R_t^* = 1) & ... & Pr(R_{t+12}^* = 6|R_t^* = 6) \end{pmatrix}$$

$$\equiv \begin{pmatrix} 0.85 & 0.10 & 0 & 0.00 & 0 & 0.01 \\ 0.05 & 0.7 & 0.07 & 0.05 & 0 & 0.02 \\ 0.04 & 0.12 & 0.8 & 0.00 & 0.05 & 0.03 \\ 0.03 & 0.01 & 0.1 & 0.85 & 0.02 & 0.04 \\ 0.02 & 0.06 & 0.01 & 0.05 & 0.93 & 0.15 \\ 0.01 & 0.01 & 0.02 & 0.05 & 0 & 0.75 \end{pmatrix}$$

$$ABILITY\_ERROR r_t^*|R_t^* \equiv \begin{pmatrix} Pr(r_t^*=1|R_t^*=1) & ... & Pr(r_t^*=1|R_t^*=6) \\ Pr(r_t^*=2|R_t^*=1) & ... & Pr(r_t^*=2|R_t^*=6) \\ . & . & . \\ Pr(r_t^*=6|R_t^*=1) & ... & Pr(r_t^*=6|R_t^*=6) \end{pmatrix}$$

$$\equiv \begin{pmatrix} 0.9 & 0.15 & 0.1 & 0 & 0 & 0 \\ 0.1 & 0.8 & 0.1 & 0 & 0 & 0 \\ 0 & 0.05 & 0.6 & 0.05 & 0 & 0.03 \\ 0 & 0 & 0.1 & 0.8 & 0.1 & 0.01 \\ 0 & 0 & 0.1 & 0.15 & 0.9 & 0.06 \\ 0 & 0 & 0 & 0 & 0 & 0.9 \end{pmatrix}$$

$$MISREPORT r_t|r_t^* \equiv \begin{pmatrix} Pr(r_t=1|r_t^*=1) & ... & Pr(r_t=1|r_t^*=6) \\ Pr(r_t=2|r_t^*=1) & ... & Pr(r_t=2|r_t^*=6) \\ . & . & . \\ Pr(r_t=6|r_t^*=1) & ... & Pr(r_t=6|r_t^*=6) \end{pmatrix}$$

$$\equiv \begin{pmatrix} 0.9 & 0.10 & 0 & 0.02 & 0 & 0.01 \\ 0.02 & 0.8 & 0.07 & 0.05 & 0 & 0.02 \\ 0.03 & 0.02 & 0.9 & 0.04 & 0 & 0.04 \\ 0.03 & 0.01 & 0 & 0.79 & 0.02 & 0.04 \\ 0.02 & 0.06 & 0.01 & 0.05 & 0.98 & 0.04 \\ 0 & 0.01 & 0.02 & 0.05 & 0 & 0.85 \end{pmatrix}$$

We sample $5,000,000$ observations and repeat 500 times, then apply our method to this simulated sample to estimate misreporting matrix and noise to check the consistency of our estimators.

We get the estimated matrix as follows:

$$MISREPORT\_Estimated \equiv \begin{pmatrix} 0.8965 & 0.0995 & 0.0021 & 0.02 & 0.0006 & 0.0105 \\ 0.0206 & 0.8006 & 0.0694 & 0.05 & 0.0015 & 0.0201 \\ 0.0297 & 0.0199 & 0.8917 & 0.0399 & 0.0025 & 0.0401 \\ 0.0299 & 0.0102 & 0.0028 & 0.7899 & 0.0199 & 0.0403 \\ 0.0197 & 0.0597 & 0.0142 & 0.0502 & 0.975 & 0.04 \\ 0.0036 & 0.0101 & 0.0198 & 0.05 & 0.0005 & 0.8491 \end{pmatrix}$$

$$MISREPORT\_std \equiv \begin{pmatrix} 0.0084 & 0.0052 & 0.0022 & 0.0018 & 0.0006 & 0.0051 \\ 0.006 & 0.0095 & 0.0046 & 0.0038 & 0.0016 & 0.0041 \\ 0.0035 & 0.0034 & 0.0099 & 0.002 & 0.0019 & 0.0017 \\ 0.0025 & 0.0038 & 0.0028 & 0.0083 & 0.0037 & 0.0064 \\ 0.0032 & 0.0065 & 0.0093 & 0.006 & 0.005 & 0.0023 \\ 0.0043 & 0.0037 & 0.0018 & 0.0051 & 0.0004 & 0.0097 \end{pmatrix}$$

$$Ability\_ERROR\_Estimated \equiv \begin{pmatrix} 0.8909 & 0.1423 & 0.0986 & 0.003 & 0.0033 & 0.0067 \\ 0.0914 & 0.8405 & 0.098 & 0.0038 & 0.0019 & 0.0073 \\ 0.0031 & 0.0045 & 0.6062 & 0.0436 & 0.0024 & 0.033 \\ 0.0046 & 0.0057 & 0.097 & 0.7764 & 0.095 & 0.0656 \\ 0.0075 & 0.0038 & 0.0969 & 0.1467 & 0.895 & 0.0561 \\ 0.0026 & 0.0031 & 0.0033 & 0.0265 & 0.0024 & 0.8313 \end{pmatrix}$$

$$Ability\_ERROR\_std \equiv \begin{pmatrix} 0.0291 & 0.0205 & 0.0093 & 0.005 & 0.0052 & 0.018 \\ 0.0233 & 0.0256 & 0.0159 & 0.0063 & 0.0033 & 0.0215 \\ 0.0052 & 0.0069 & 0.017 & 0.0157 & 0.0036 & 0.0319 \\ 0.0076 & 0.009 & 0.0084 & 0.0463 & 0.0083 & 0.0599 \\ 0.0102 & 0.006 & 0.0129 & 0.027 & 0.0104 & 0.0206 \\ 0.0043 & 0.0051 & 0.0038 & 0.0362 & 0.0036 & 0.0913 \end{pmatrix}$$

The t-tests could be conducted for cell-to-cell difference between true and estimated misreporting matrix. Comparing the true and estimated misreporting matrix, we can see that the difference between these two matrices is reasonably small and all cells are statistically insignificant. In untabulated analyses, the difference between true misreporting matrix and estimated misreporting matrix becomes smaller when the simulation sample size increase. Therefore, our method is able to correctly estimate the misreporting probability.

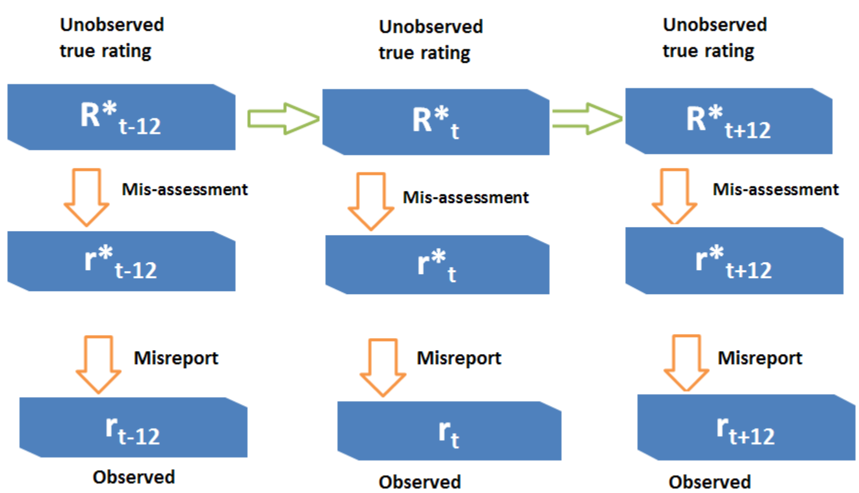

Figure 1: Model setup

This figure shows the model setup. The R^*s are the unobserved true rating. The r^*s are the unobserved internal rating by CRAs. The rs are the reported rating by CRAs. Since CRAs normally reevaluate the rating once a year unless significant corporate events occur, such as merger and acquisition, we use data one year apart to mitigate the influence of past reported rating on future reported rating when conditional on observed rating. This is to alleviate the concern of time series correlation of the reported ratings. Therefore, our model allow the autocorrelation between credit rating to be up to 11 time steps.

Table 1: Yearly distribution of the ratings

This table displays the S&P Long-Term Issuer credit rating distribution across the entire sample. We aggregate the monthly data into yearly observation. There are 21 different levels of rating by the rating agency and we regroup them into 6 categories: All the AAA level rating to be category 1; All the AA level rating to be category 2; All the A level rating to be category 3; category 4 includes all the triple B rating, i.e BBB+, BBB,BBB-; category 5 includes all the double B rating, i.e BB+, BB,BB-; and category 6 consists of all rating left.

				Rating category			
year	AAA	AA	A	BBB	BB&B	Below B	Total
1986	664	3,694	8,171	10,032	20,217	2,027	44,805
1987	690	3,589	7,936	9,941	21,003	2,121	45,280
1988	663	3,381	7,934	9,855	20,715	2,079	44,627
1989	705	3,015	8,123	9,885	20,288	2,056	44,072
1990	690	2,989	7,754	10,013	19,423	1,915	42,784
1991	668	2,826	7,629	10,071	18,747	1,965	41,906
1992	634	2,821	7,659	10,025	18,771	1,868	41,778
1993	582	2,701	7,809	10,216	19,059	1,600	41,967
1994	560	2,652	7,743	10,332	19,432	1,315	42,034
1995	552	2,513	7,823	10,476	19,090	1,427	41,881
1996	571	2,427	7,903	10,347	19,339	1,372	41,959
1997	541	2,405	7,849	10,725	19,439	1,247	42,206
1998	505	2,357	7,665	10,819	19,211	1,216	41,773
1999	474	2,321	7,421	10,740	18,856	1,627	41,439
2000	409	2,080	7,144	10,443	18,305	1,843	40,224
2001	356	1,725	6,845	10,180	17,371	2,183	38,660
2002	314	1,461	6,249	10,018	16,620	2,165	36,827
2003	277	1,209	5,825	9,579	16,454	2,003	35,347
2004	255	1,183	5,460	9,531	16,444	1,492	34,365
2005	229	1,188	5,260	9,379	15,684	1,171	32,911
2006	223	1,215	5,011	8,945	14,888	950	31,232
2007	212	1,263	4,673	8,535	13,958	751	29,392
2008	198	1,220	4,438	8,330	12,665	813	27,664
2009	154	1,000	4,057	8,259	11,315	1,599	26,384
2010	128	873	4,038	8,056	11,121	899	25,115
2011	128	788	4,072	7,925	10,205	668	23,786
Total	11,382	54,896	172,491	252,657	448,620	40,372	980,418

Table 2: One year transition probability matrix for credit ratings— entire sample

This table displays estimation of one year transition probability matrix for reported and true ratings for whole sample. The sample includes all firms with a Long-Term Issuer Credit Rating from S&P and the sample period is from 1986 to 2011. The true rating transition matrix is estimated using method proposed in the paper. There are 21 different levels of rating by the rating agency and we regroup them into 6 categories: All the AAA level rating to be category 1; All the AA level rating to be category 2; All the A level rating to be category 3; category 4 includes all the triple B rating, i.e BBB+, BBB,BBB-; category 5 includes all the double B rating, i.e BB+, BB,BB-; and category 6 consists of all rating left.

Panel A: The transition matrix using observed ratings

		Rating category for 12 months later					
		AAA	AA	A	BBB	BB&B	Below B
Current rating	AAA	0.9005	0.0838	0.0109	0.0002	0.0027	0.00001
	AA	0.0102	0.858	0.118	0.0105	0.0024	0.0001
	A	0.00008	0.0236	0.876	0.0869	0.0108	0.0014
	BBB	0.002	0.0021	0.0474	0.875	0.0703	0.0042
	BB&B	0.0001	0.0004	0.00037	0.0390	0.918	0.0384
	Below B	0.0	0.0003	0.00066	0.0273	0.384	0.581

Panel B: The estimated transition matrix after correction of misreporting

		Rating category for 12 months later					
		AAA	AA	A	BBB	BB&B	Below B
Current rating	AAA	0.943	0.052	0	0	0.005	0
	AA	0.002	0.91	0.087	0	0.002	0
	A	0	0.012	0.923	0.065	0	0.001
	BBB	0	0.001	0.028	0.919	0.05	0.002
	BB&B	0	0	0.001	0.029	0.929	0.041
	Below B	0	0	0	0.006	0.179	0.815

Table 3: Misreporting probability for each rating group — entire sample

This table displays estimation of misreporting probability for entire sample. The sample includes all firms with a Long-Term Issuer Credit Rating from S&P and the sample period is from 1986 to 2011. There are 21 different levels of rating by the rating agency and we regroup them into 6 categories: All the AAA level rating to be category 1; All the AA level rating to be category 2; All the A level rating to be category 3; category 4 includes all the triple B rating, i.e BBB+, BBB,BBB-; category 5 includes all the double B rating, i.e BB+, BB,BB-; and category 6 consists of all rating left. The diagonal of matrix stands for the trustfully reporting probability and the cells in upper triangle stand for the inflation probability. The cells in lower triable stand for the deflation probability. The * stands for statistical difference between this matrix and identity matrix (cell to cell). ***significant at the 1% level, **significant at the 5% level, *significant at the 10% level. .

		True Rating					
		AAA	AA	A	BBB	BB&B	Below B
Reported rating	AAA	0.969**	0.004	0.001	0	0	0
	AA	0.02**	0.972***	0	0	0	0
	A	0.008	0.018***	0.988	0.003	0	0
	BBB	0.002	0.006	0.002	0.995	0.006	0.001
	BB&B	0.001	0.001	0.008	0.001	0.994	0.065***
	Below B	0	0	0	0.001	0	0.934***
Total misreporting Prob.		0.031***	0.028***	0.012	0.005	0.006	0.066***

Table 4: Misreporting behavior difference across industries

This table displays estimation of misreporting probability for each rating groups. The sample includes all firms with a Long-Term Issuer Credit Rating from S&P and the sample period is from 1986 to 2011. The financial firms are defined as 2-digit GICS equal to 40. The energy firms are defined as 2-digit GICS equal to 10. The financial firms are defined as 2-digit GICS equal to 30.The diagonal of matrix stands for the trustfully reporting probability and the cells in upper triangle stand for the inflation probability. . The cells in lower triable stand for the deflation probability. The * stands for statistical difference between this matrix and identity matrix (cell to cell). ***significant at the 1% level, **significant at the 5% level, *significant at the 10% level.

Panel A: Misreporting matrix for financial industry

		True Rating					
		AAA	AA	A	BBB	BB&B	Below B
Reported rating	AAA	0.957***	0	0	0	0	0
	AA	0.04***	0.977***	0.001	0	0	0.355***
	A	0.002***	0.019***	0.992***	0.002	0.002	0
	BBB	0.001***	0.004***	0.003***	0.998	0.094***	0.002
	BB&B	0	0	0.004	0	0.903***	0.021
	Below B	0	0	0	0	0.001	0.621***
Total misreporting Prob.		0.043***	0.023***	0.008***	0.002	0.097***	0.379***

Panel B: Misreporting matrix for energy industry

		True Rating					
		AAA	AA	A	BBB	BB&B	Below B
Reported rating	AAA	0.996	0	0	0	0	0
	AA	0.004	0.936**	0	0	0	0
	A	0	0.047**	0.99*	0.002	0.001	0.001
	BBB	0	0	0.009*	0.996*	0.007	0.004*
	BB&B	0	0.004	0	0.001**	0.985	0.076
	Below B	0	0.013	0	0	0.007	0.92
Total misreporting Prob.		0.004	0.064	0.01	0.004	0.015	0.08

Panel C: Misreporting matrix for consumer staples industry

		True Rating					
		AAA	AA	A	BBB	BB&B	Below B
Reported rating	AAA	0.907***	0	0.019***	0	0	0
	AA	0.069***	0.938***	0	0.001	0	0
	A	0.02	0.043***	0.964***	0	0	0
	BBB	0.004	0.019	0.018***	0.981***	0.003	0.001
	BB&B	0	0	0	0.018***	0.996	0.348***
	Below B	0	0	0	0	0.001	0.651***
Total misreporting Prob.		0.093***	0.062***	0.036***	0.019***	0.004	0.349***

Table 5: Misreporting behavior across business cycle

This table displays estimation of misreporting probability for each rating groups. The sample includes all firms with a Long-Term Issuer Credit Rating from S&P and the sample period is from 1986 to 2011. There are 21 different levels of rating by the rating agency and we regroup them into 6 categories: All the AAA level rating to be category 1; All the AA level rating to be category 2; All the A level rating to be category 3; category 4 includes all the triple B rating, i.e BBB+, BBB,BBB-; category 5 includes all the double B rating, i.e BB+, BB,BB-; and category 6 consists of all rating left. We divide the sample into tercile base on real GDP growth rate. The years in highest growth rate tercile are defined as economic boom and the years with lowest growth rate are defined as economic recession. . The diagonal of matrix stands for the trustfully reporting probability and the cells in upper triangle stand for the inflation probability. The cells in lower triable stand for the deflation probability. The * stands for statistical difference between this matrix and identity matrix (cell to cell). ***significant at the 1% level, **significant at the 5% level, *significant at the 10% level.

Panel A Misreporting behavior during economic boom

		True Rating					
		AAA	AA	A	BBB	BB&B	Below B
Reported rating	AAA	0.879***	0.002***	0	0	0	0.001
	AA	0.09***	0.894***	0.026***	0.002	0.001	0.077**
	A	0.027***	0.093***	0.931***	0.022***	0.001	0.067**
	BBB	0.003	0.009***	0.037***	0.932***	0.017***	0.014
	BB&B	0.001	0.002	0.006	0.041***	0.97***	0.194***
	Below B	0	0	0	0.003	0.011***	0.648***
Total misreporting Prob.		0.121***	0.106***	0.069***	0.068***	0.03***	0.352***

Panel B Misreporting behavior during economic recession

		True Rating					
		AAA	AA	A	BBB	BB&B	Below B
Reported rating	AAA	0.864***	0.002	0	0	0	0
	AA	0.121***	0.92***	0.016***	0.001	0	0.001
	A	0.013	0.065***	0.96***	0.025***	0.001	0.004
	BBB	0.002	0.011***	0.018***	0.941***	0.013**	0.012
	BB&B	0	0.001	0.005	0.03***	0.967***	0.258***
	Below B	0	0.001	0.001	0.003	0.018**	0.726***
Total misreporting Prob.		0.136***	0.08***	0.04***	0.059***	0.033**	0.274***

Table 6: Misreporting probability for financial firms (exclude current financial crisis)

This table displays estimation of misreporting probability for each rating groups. The sample includes all firms with a Long-Term Issuer Credit Rating from S&P and the sample period is from 1986 to 2011. There are 21 different levels of rating by the rating agency and we regroup them into 6 categories: All the AAA level rating to be category 1; All the AA level rating to be category 2; All the A level rating to be category 3; category 4 includes all the triple B rating, i.e BBB+, BBB,BBB-; category 5 includes all the double B rating, i.e BB+, BB,BB-; and category 6 consists of all rating left.The financial firms are defined as 2-digit GICS equal to 40.The diagonal of matrix stands for the trustfully reporting probability and the cells in upper triangle stand for the inflation probability. . The cells in lower triable stand for the deflation probability. The * stands for statistical difference between this matrix and identity matrix (cell to cell). ***significant at the 1% level, **significant at the 5% level, *significant at the 10% level.

		True Rating					
		AAA	AA	A	BBB	BB&B	Below B
Reported rating	AAA	0.938***	0.001	0.001	0	0.001	0
	AA	0.052***	0.953***	0.007***	0	0.001	0
	A	0.006	0.042***	0.976	0.015***	0.004	0.005
	BBB	0.004	0.005	0.009***	0.969***	0.018	0.349***
	BB&B	0	0.001	0.007	0.015***	0.976	0.046
	Below B	0	0	0	0.001	0.001	0.599***
Total misreporting Prob.		0.062***	0.047***	0.024	0.031***	0.024	0.401***